2. Vector Algebra
Vector basics

Vector: $\mathbf{A}, \mathbf{\bar{A}}$
Magnitude: $|\mathbf{A}|, A$
Scalar: $p, \phi$

Types
– Polar vector
  •
– Axial vector
  •
– Unit vector
  •
Vector Algebra

• Addition

\[ A + B = \]

• Dot, or scalar, product

\[ A \cdot B = AB \cos \theta \]

• E.g. Work=\( F \cdot s \)

• Flow rate through \( dA = V \cdot dA \) or \( V \cdot ndA \)

• \( A \cdot B = B \cdot A \)

\[ A \cdot A = A^2 \]

\[ A \cdot B = 0 \text{ if perpendicular} \]
Vector Algebra

- Cross, or vector, product
  \[ \mathbf{A} \times \mathbf{B} = \mathbf{A}B \sin \theta \mathbf{e} \]

- \[ \mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} \]
- \[ \mathbf{A} \times \mathbf{A} = 0 \]
- \[ \mathbf{A} \times \mathbf{B} = 0 \text{ if } \mathbf{A} \text{ and } \mathbf{B} \text{ parallel} \]
Vector Algebra – Triple Products

1. \((A \cdot B)C = (B \cdot A)C\)

2. Mixed product \(A \cdot B \times C\)
   - Volume of parallelepiped bordered by \(A, B, C\)
   - May be cyclically permuted
     \(A \cdot B \times C = C \cdot A \times B = B \cdot C \times A\)
   - Acyclic permutation changes sign
     \(A \cdot B \times C = -B \cdot A \times C\) etc.

3. Vector triple product
   - \(A \times (B \times C)\) = Vector in plane of \(B\) and \(C\)
     = ...
Cartesian Coordinates

- Coordinates $x$, $y$, $z$
- Unit vectors $i$, $j$, $k$ (in directions of increasing coordinates) are constant
- Position vector $\mathbf{r} = \ldots$
- Vector components $\mathbf{F} = \ldots$

Components same regardless of location of vector
Cylindrical Coordinates

- Coordinates $r$, $\theta$, $z$
- Unit vectors $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_z$ (in directions of increasing coordinates)
- Position vector
  $$ \mathbf{R} = $$
- Vector components
  $$ \mathbf{F} = $$

Components not constant, even if vector is constant
Spherical Coordinates

- Coordinates \( r, \theta, \phi \)
- Unit vectors \( \mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\phi \) (in directions of increasing coordinates)
- Position vector \( \mathbf{r} = \)
- Vector components \( \mathbf{F} = \)
Vector Algebra in Components

\[ \mathbf{A} \cdot \mathbf{B} = A_1 B_1 + A_2 B_2 + A_3 B_3 \]

\[ \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} \]
Concept of Differential Change In a Vector. The Vector Field.

Scalar field \( \phi = \phi(r, t) \)
Vector field \( \mathbf{V} = \mathbf{V}(r, t) \)

Differential change in vector
- Change in
- Change in
Change in Unit Vectors – Cylindrical System
Change in Unit Vectors – Spherical System

\[ \text{change in } e_r = d\theta e_\theta + d\phi \sin \theta e_\phi \]
\[ \text{change in } e_\theta = -d\theta e_r + d\phi \cos \theta e_\phi \]
\[ \text{change in } e_\phi = -d\phi \sin \theta e_r - d\phi \cos \theta e_\theta \]

See “Formulae for Vector Algebra and Calculus”
Example

The position of fluid particle moving in a flow varies with time. Working in different coordinate systems write down expressions for the position and, by differentiation, the velocity vectors.

**Cartesian System**

**Cylindrical System**

... This is an example of the calculus of vectors with respect to time.
Vector Calculus w.r.t. Time

• Since *any* vector may be decomposed into scalar components, calculus w.r.t. time, only involves *scalar* calculus of the components.

\[
\frac{\partial (A + B)}{\partial t} = \frac{\partial A}{\partial t} + \frac{\partial B}{\partial t}
\]

\[
\frac{\partial (A \cdot B)}{\partial t} = A \frac{\partial B}{\partial t} + \frac{\partial A}{\partial t} \cdot B
\]

\[
\frac{\partial (A \times B)}{\partial t} = A \times \frac{\partial B}{\partial t} + \frac{\partial A}{\partial t} \times B
\]

\[
\int (A + B)dt = \int A dt + \int B dt
\]