



C A L T E C H
Control & Dynamical Systems

A Statistical Theory of Transition and Collision Probabilities

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Overview

■ *Motivation*

- Planetary science
- Comet motion, collisions
- Chaotic dynamics, intermittency

■ *Transport theory*

- Restricted three-body problem
- Predictions of theory
- Comparison with observational data?

Motivation

■ *Planetary science*

- Current astrophysical interest in understanding the transport and origin of solar system material:
 - How likely is **Oterma**-like resonance transition?
 - How likely is **Shoemaker-Levy 9**-type collision with Jupiter?
 - or an **asteroid collision** with Earth?
 - How does impact ejecta get from Mars to Earth?
- **Statistical methods** applied to the three-body problem may provide a first-order answer.
- The “interaction” of several three-body systems increases the complexity.

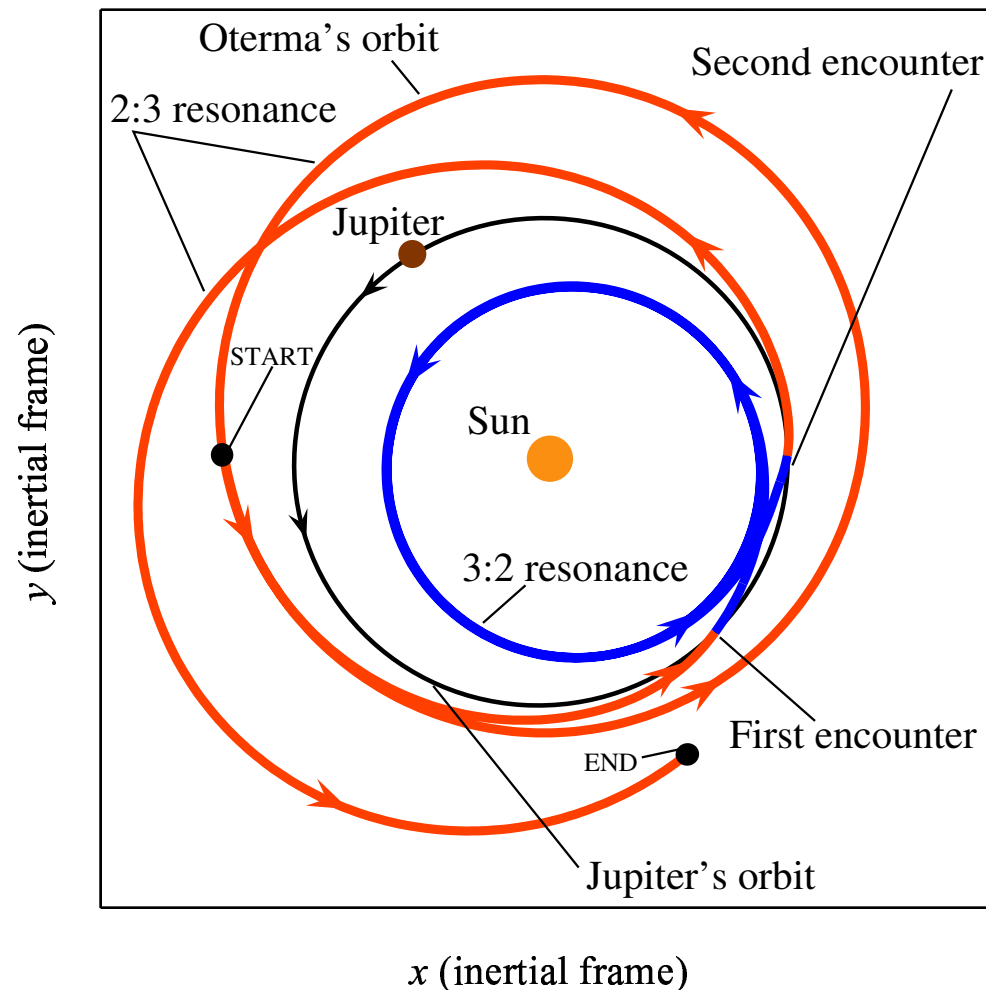
Jupiter Family Comets

■ *Physical example of intermittency*

- We consider the **historical record** of the comet **Oterma** from 1910 to 1980
 - first in an inertial frame
 - then in a rotating frame
 - a special case of pattern evocation
- Similar pictures exist for many other comets

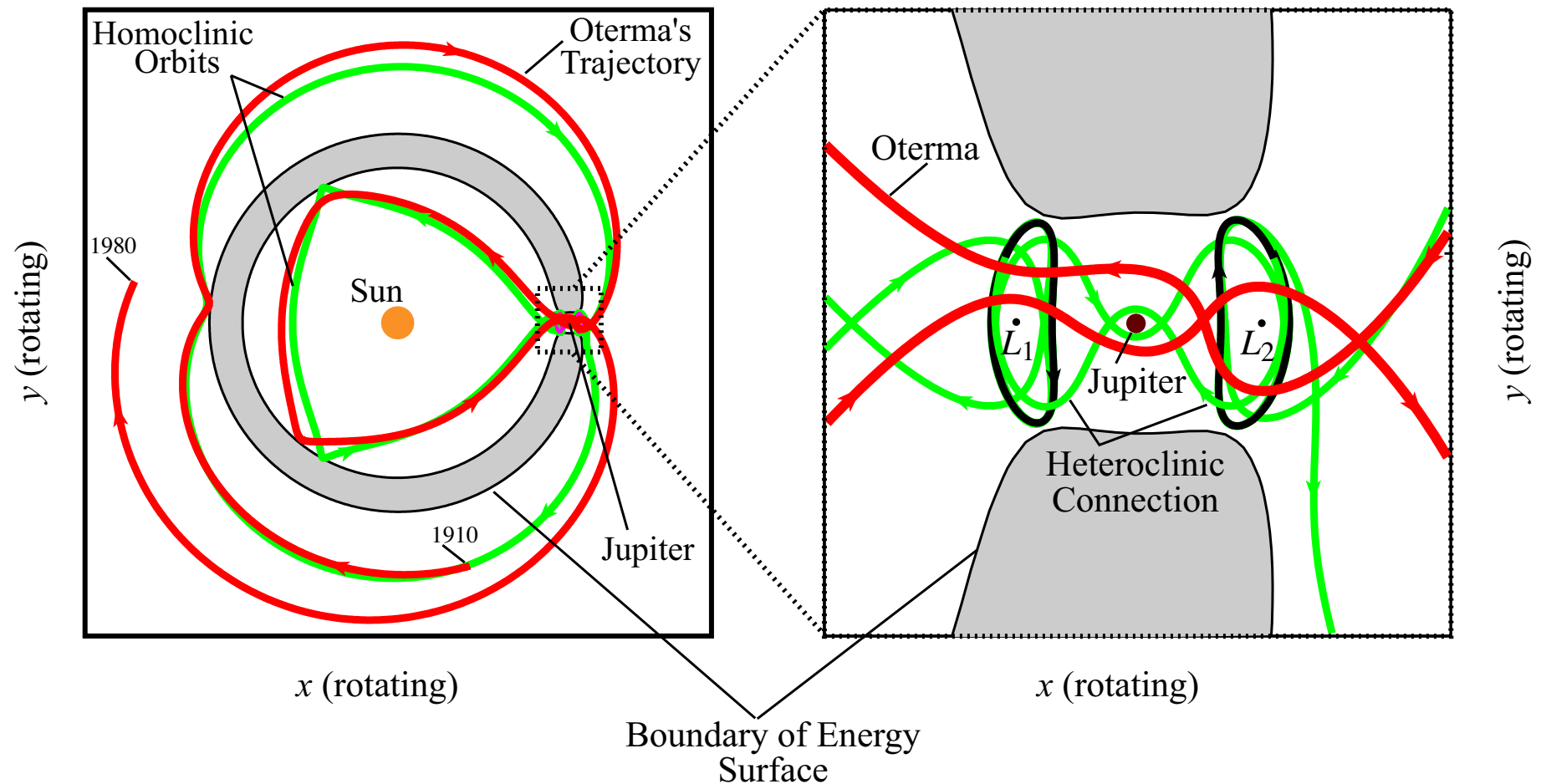
Jupiter Family Comets

- Rapid transition: outside to inside Jupiter's orbit.
 - Captured temporarily by Jupiter during transition.
 - Exterior (2:3 resonance) to interior (3:2 resonance).



Viewed in Rotating Frame

- **Oterma's orbit** in rotating frame with some invariant manifolds of the 3-body problem superimposed.

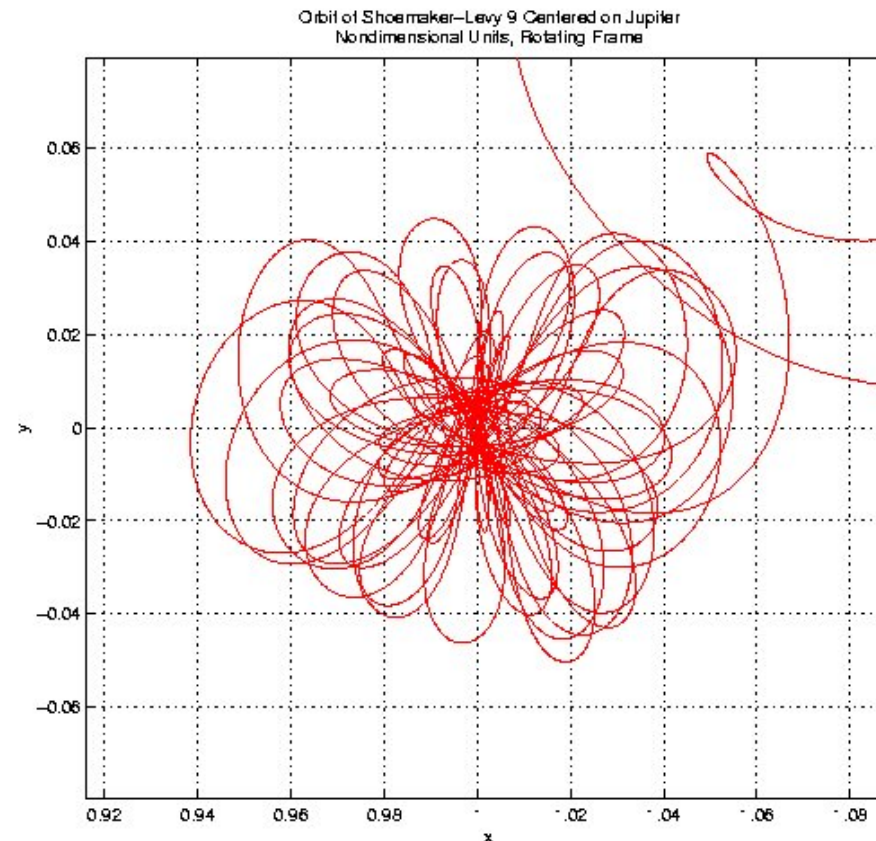


Viewed in Rotating Frame

Oterma - Rotating Frame

Collisions with Jupiter

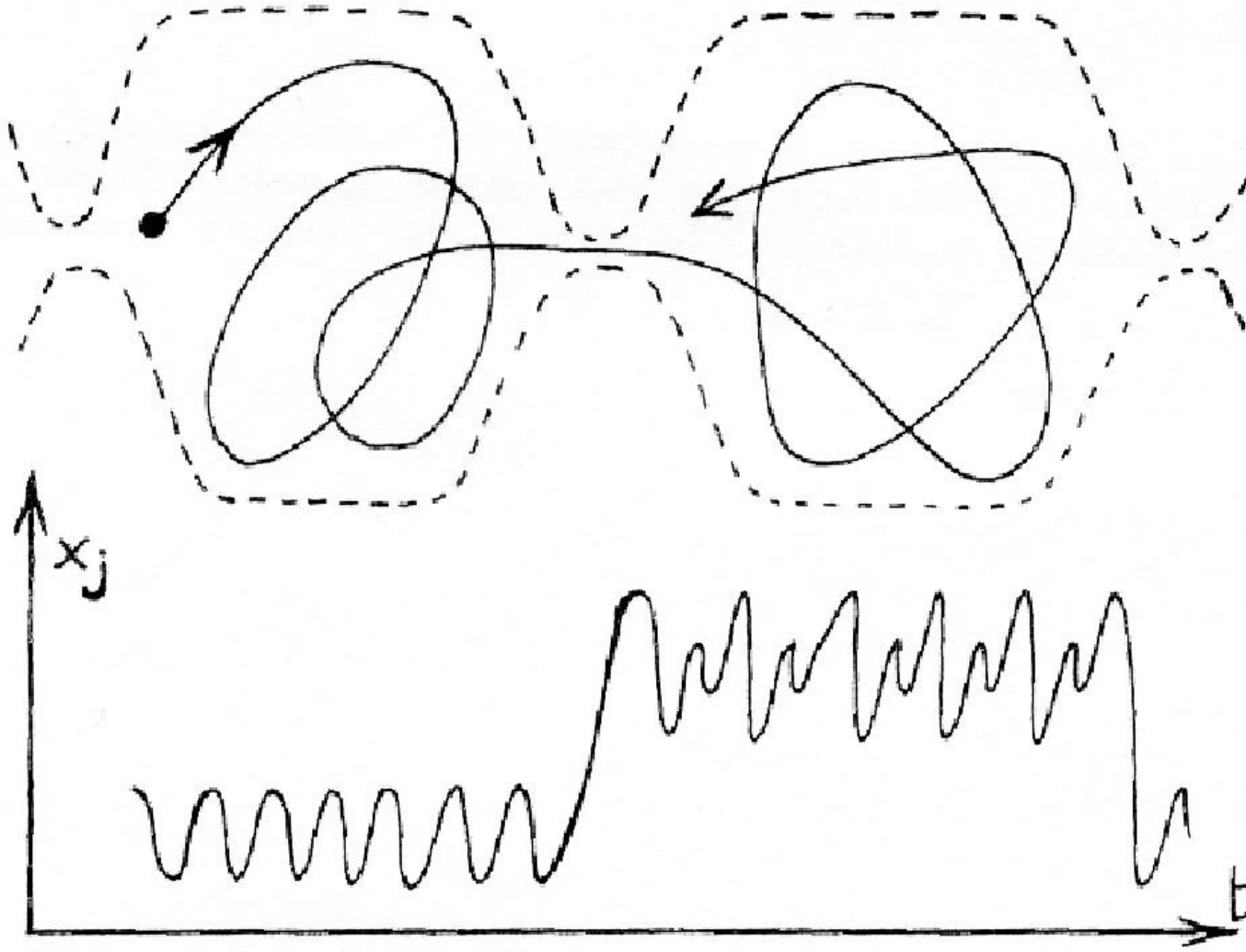
- Shoemaker Levy-9: similar energy to Oterma
 - Temporary capture and collision; came through L1 or L2



Possible *Shoemaker-Levy 9* orbit seen in rotating frame (Chodas, 2000)

Chaotic Dynamics

Transport through a **bottleneck** in phase space; intermittency



Transport Theory

■ *Chaotic dynamics*

\implies *statistical methods*

■ *Transport theory*

□ **Ensembles** of phase space trajectories

- How long or likely to move from one region to another?
- Determine transition probabilities

□ Applications:

- Comet and asteroid collision probabilities, resonance transition probabilities, transport rates

Transport Theory

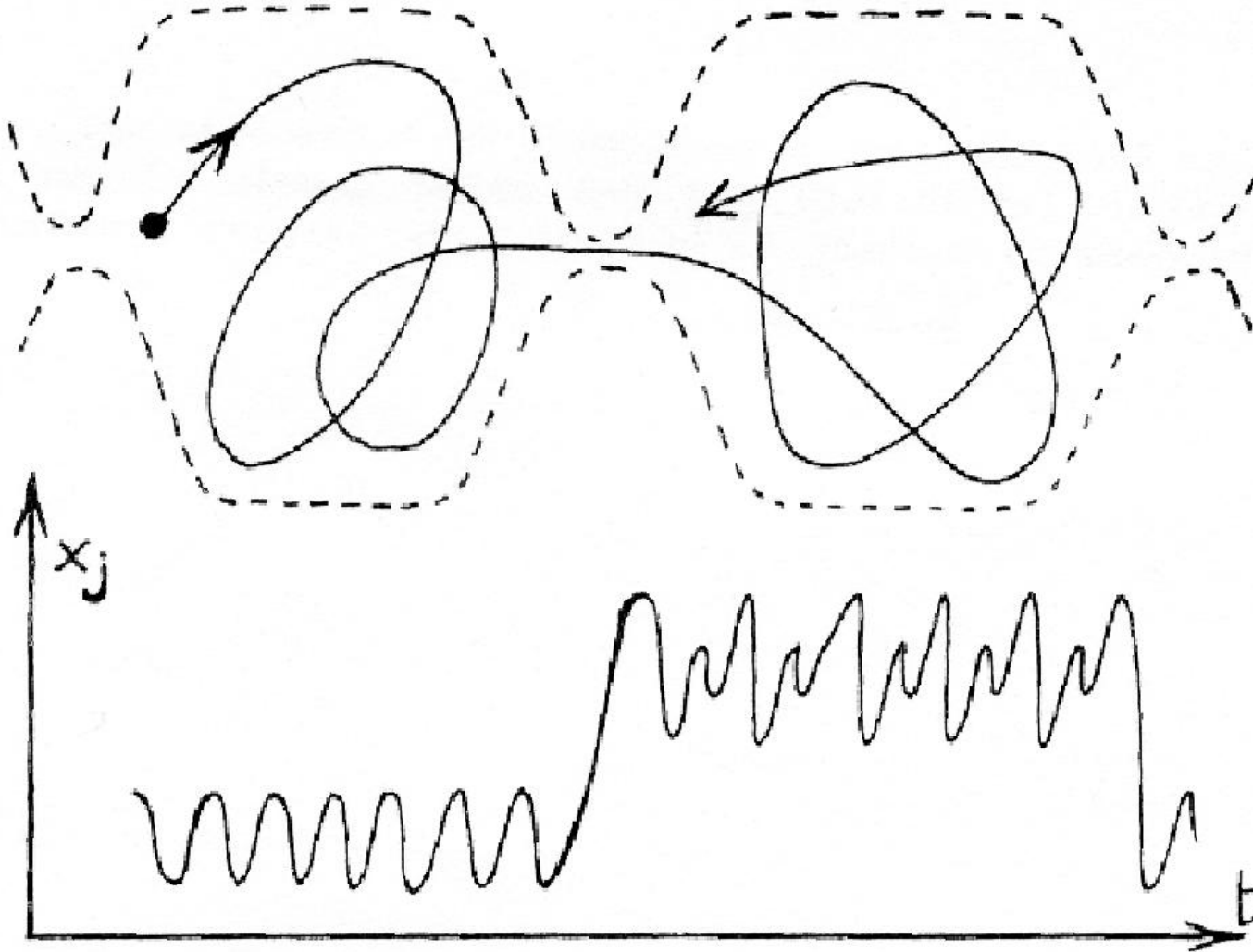
■ *Transport in the solar system*

- For objects of interest
 - e.g., Jupiter family comets, near-Earth asteroids, dust
- **Identify phase space objects** governing transport
- View N -body as multiple restricted 3-body problems
- Consider stable/unstable manifolds of bounded orbits associated with **libration points**
 - e.g, planar Lyapunov orbits
- Use these to **compute statistical quantities**
 - e.g., probability of resonance transition, escape rates

Partition the Phase Space

Region A

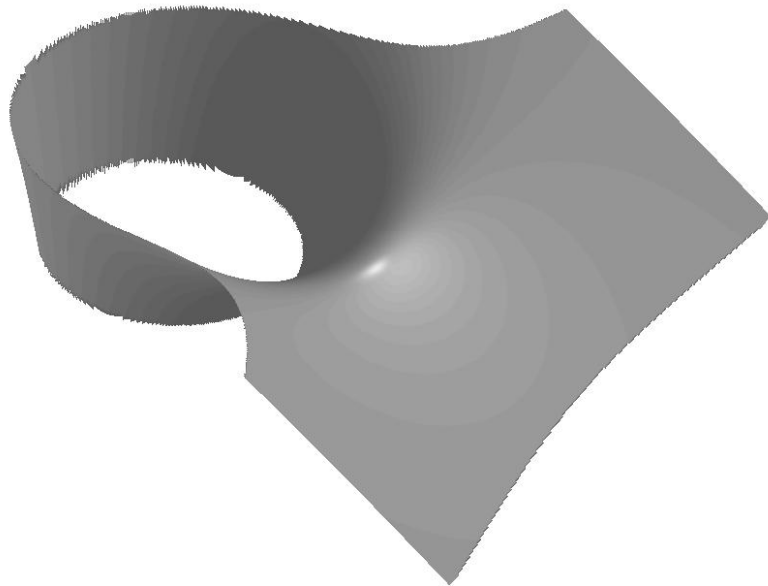
Region B



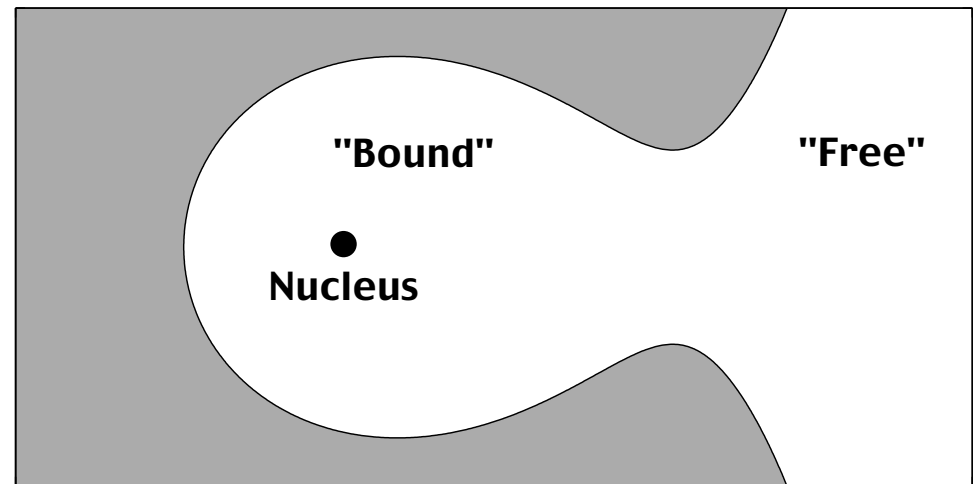
Partition the Phase Space

■ *Systems with potential barriers*

- Electron near a nucleus with crossed electric and magnetic fields
 - See Jaffé, Farrelly, and Uzer [1999]



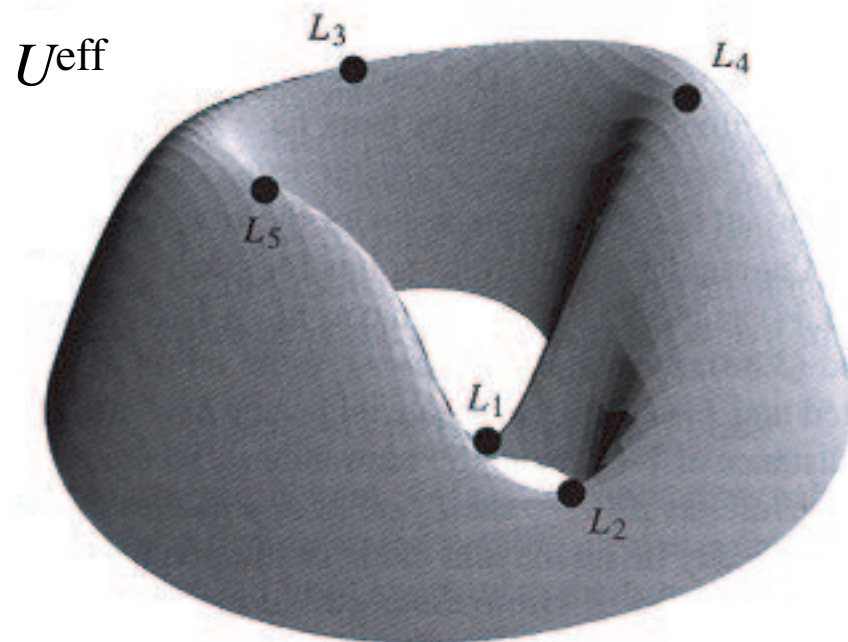
Potential



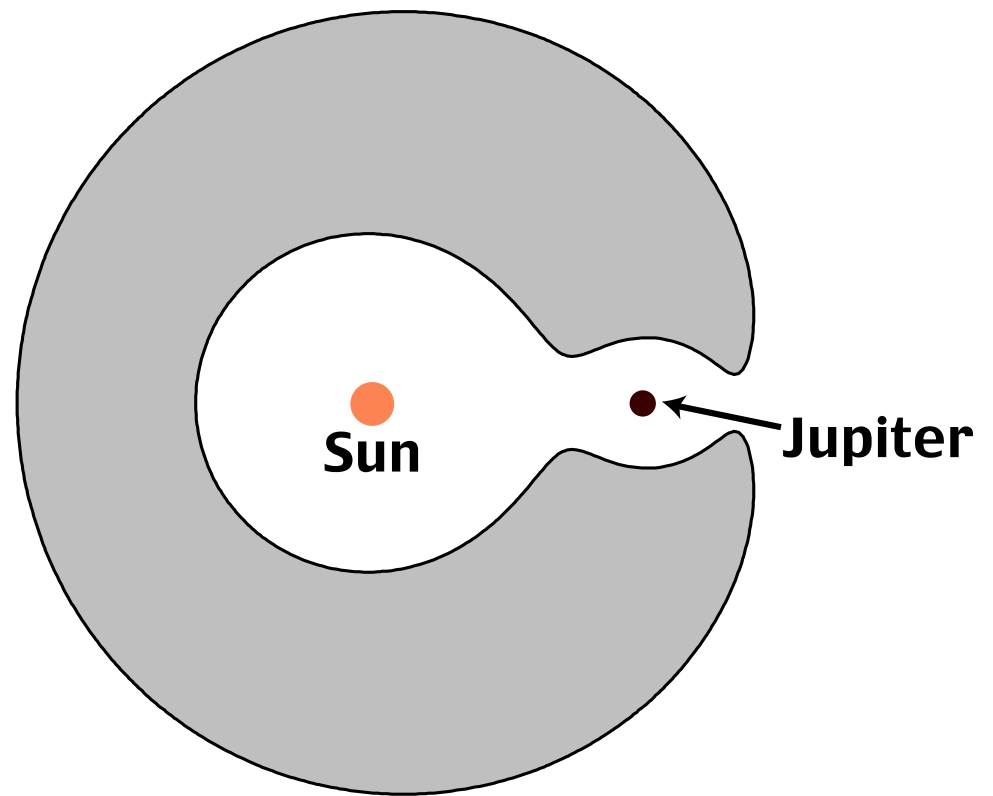
Configuration Space

Partition the Phase Space

- Comet near the Sun and Jupiter
 - Some behavior similar to electron!



Potential



Configuration Space

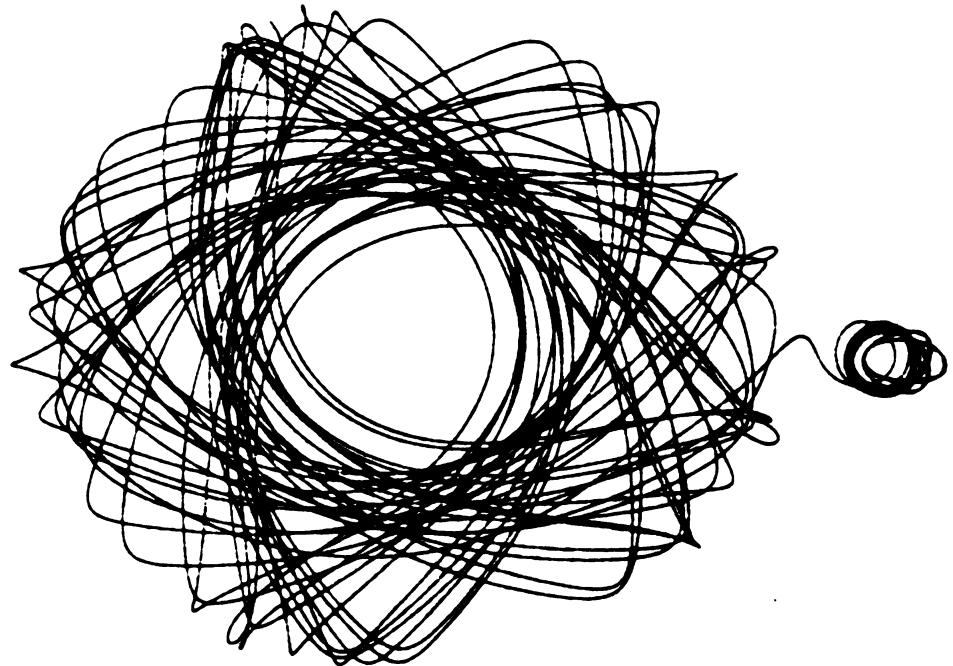
Partition the Phase Space

■ *Partition is specific to problem*

- We desire a way of describing dynamical boundaries that represent the “**frontier**” between qualitatively different types of behavior

■ *Example: motion of a comet*

- motion around the Sun
- motion around Jupiter



Statement of Problem

- Following Wiggins [1992]:
- Suppose we study the motion on a manifold \mathcal{M}
- Suppose \mathcal{M} is partitioned into disjoint regions

$$R_i, i = 1, \dots, N_R,$$

such that

$$\mathcal{M} = \bigcup_{i=1}^{N_R} R_i.$$

- At $t = 0$, **region** R_i is **uniformly covered** with **species** S_i
- Thus, species type of a point indicates the region in which it was **located initially**

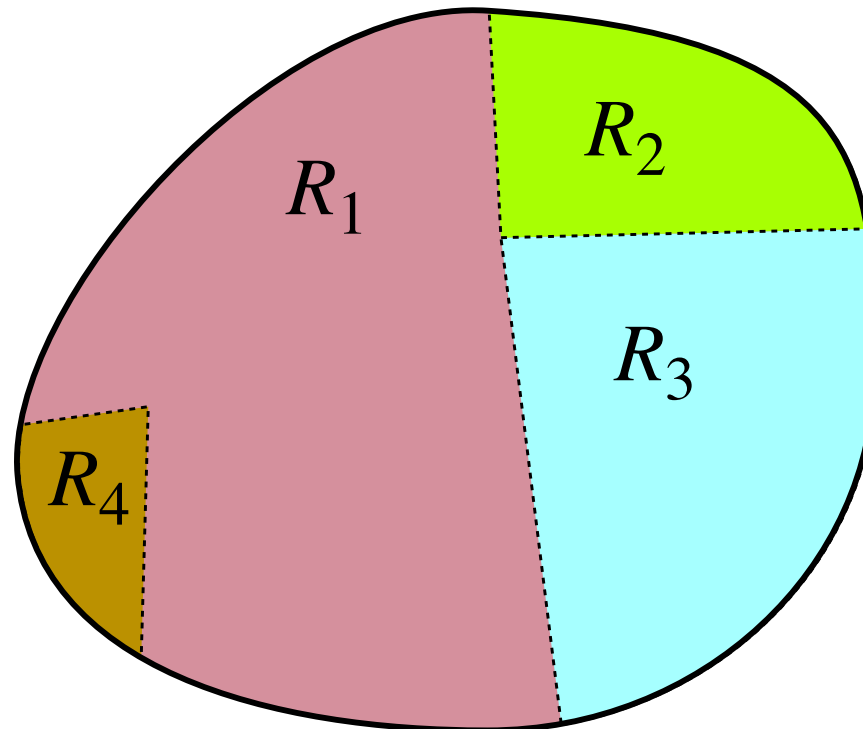
Statement of Problem

- Statement of the transport problem:

Describe the distribution of species

$S_i, i = 1, \dots, N_R$, throughout the regions

$R_j, j = 1, \dots, N_R$, for any time $t > 0$.

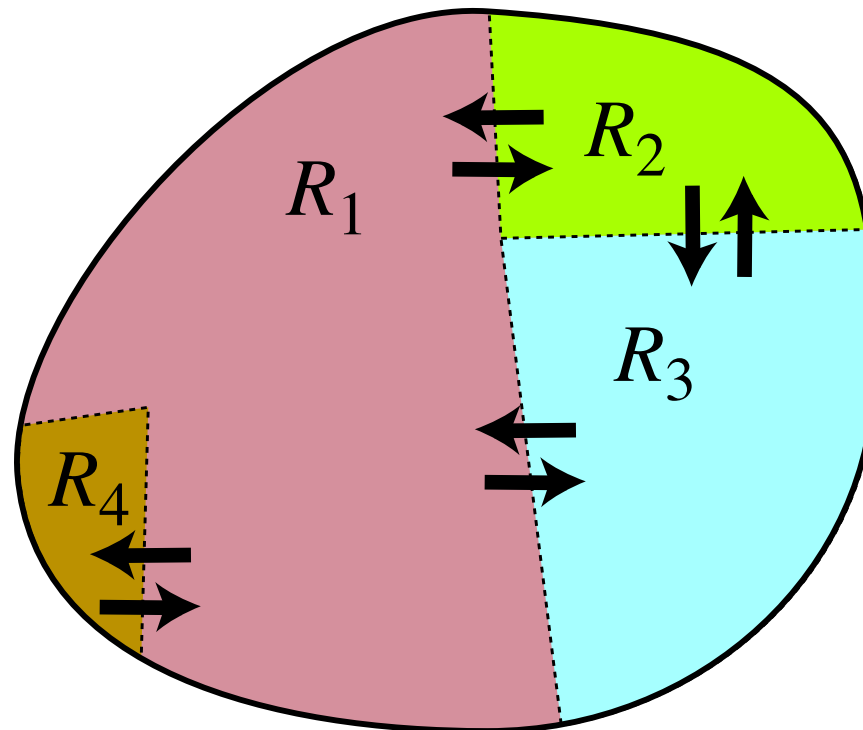


Statement of Problem

- Some quantities we would like to compute are:

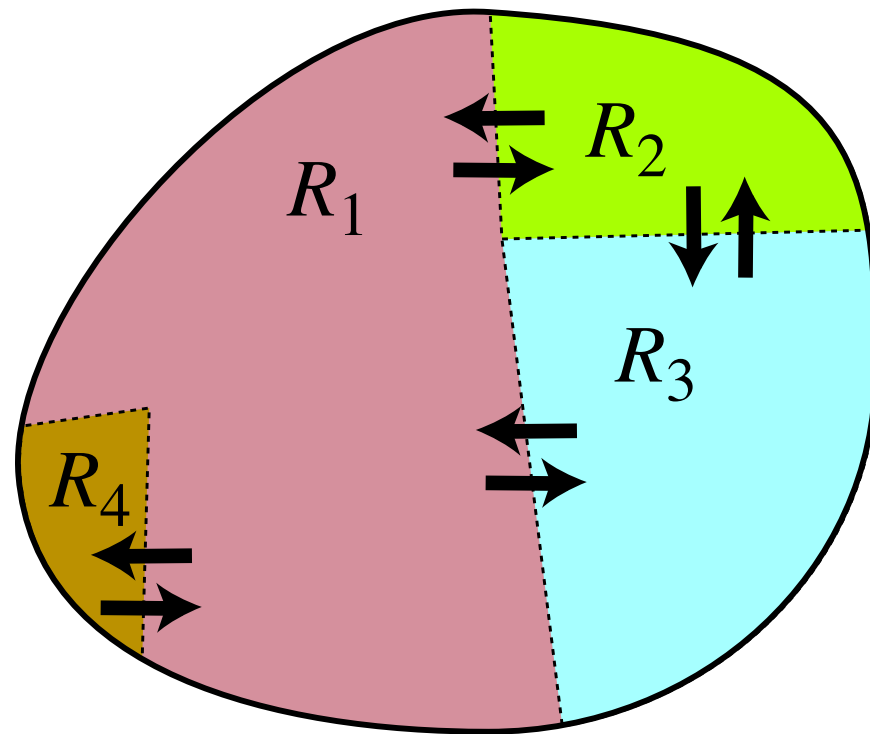
$T_{i,j}(t)$ = amount of species S_i contained in region R_j

$F_{i,j}(t) = \frac{dT_{i,j}}{dt}(t)$ = flux of species S_i into region R_j



Probabilities

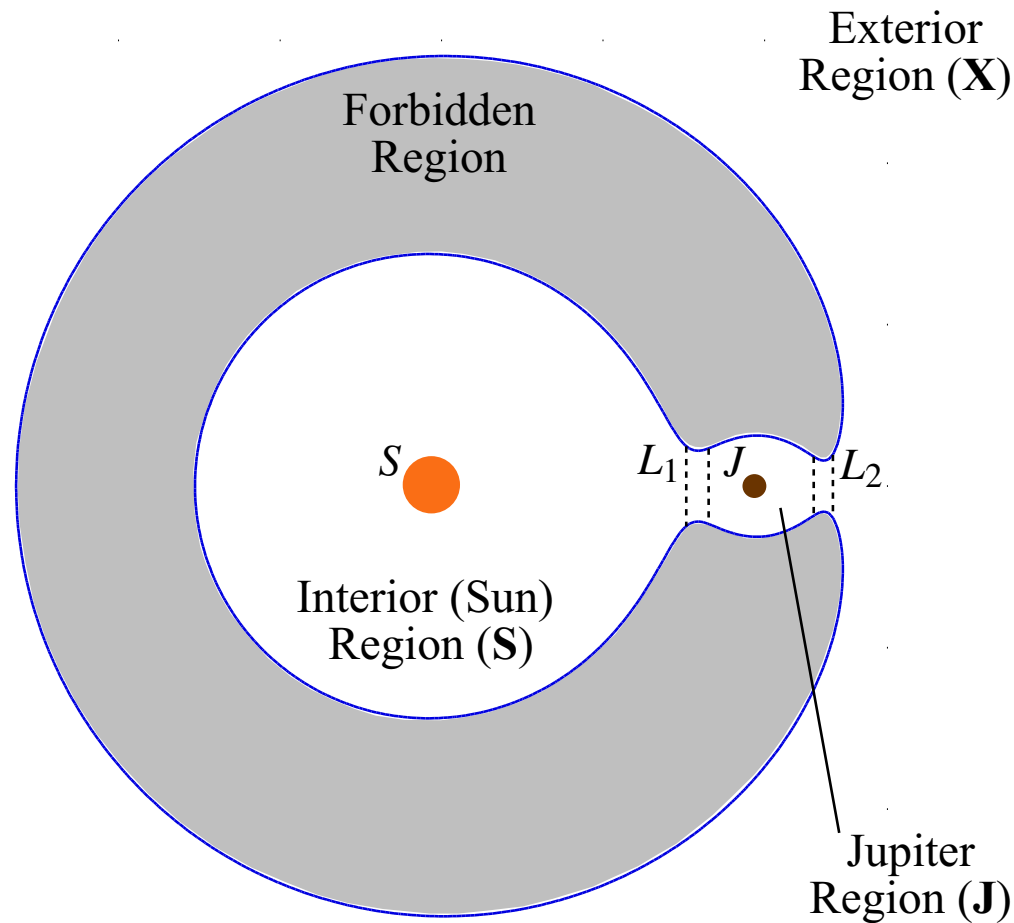
- For some problems, probability more relevant
 - e.g., probability = 0 implies event should not occur
- Test this on celestial mechanics problems of interest



Restricted 3-Body Prob.

■ *Planar circular case*

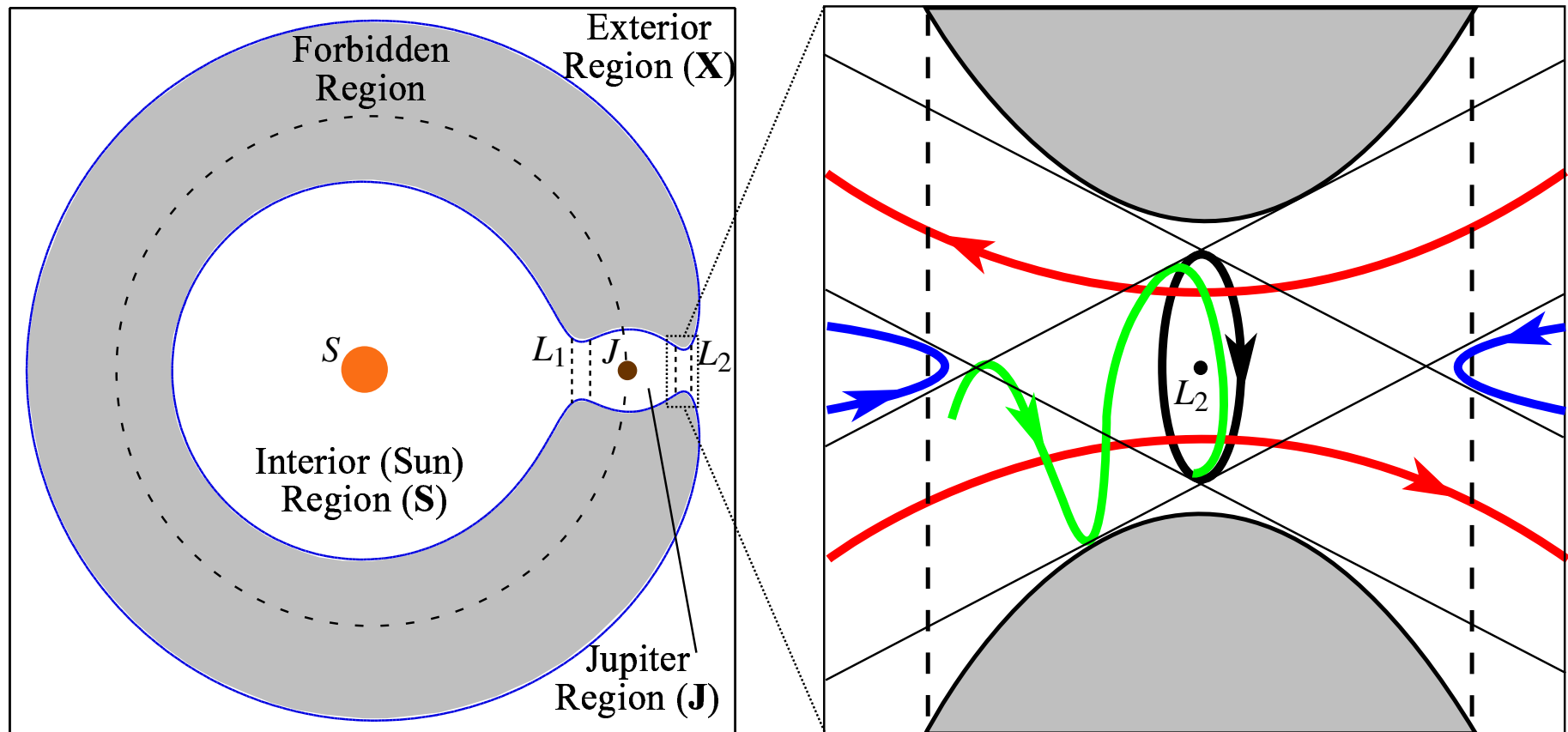
□ Partition the energy surface: **S, J, X** regions



Position Space Projection

Equilibrium Region

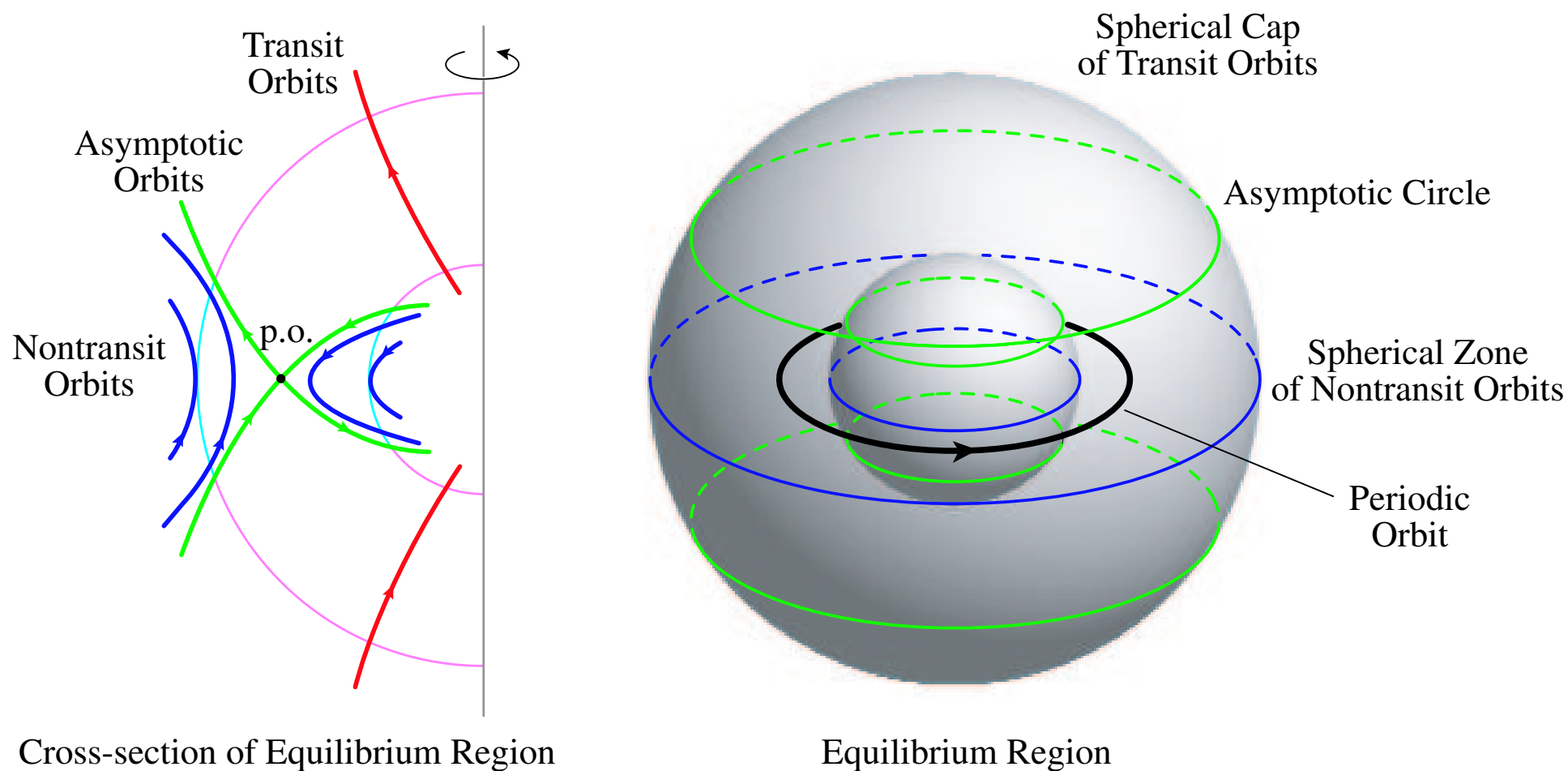
- *Look at motion near the potential barrier, i.e. the equilibrium region*



Position Space Projection

Local Dynamics

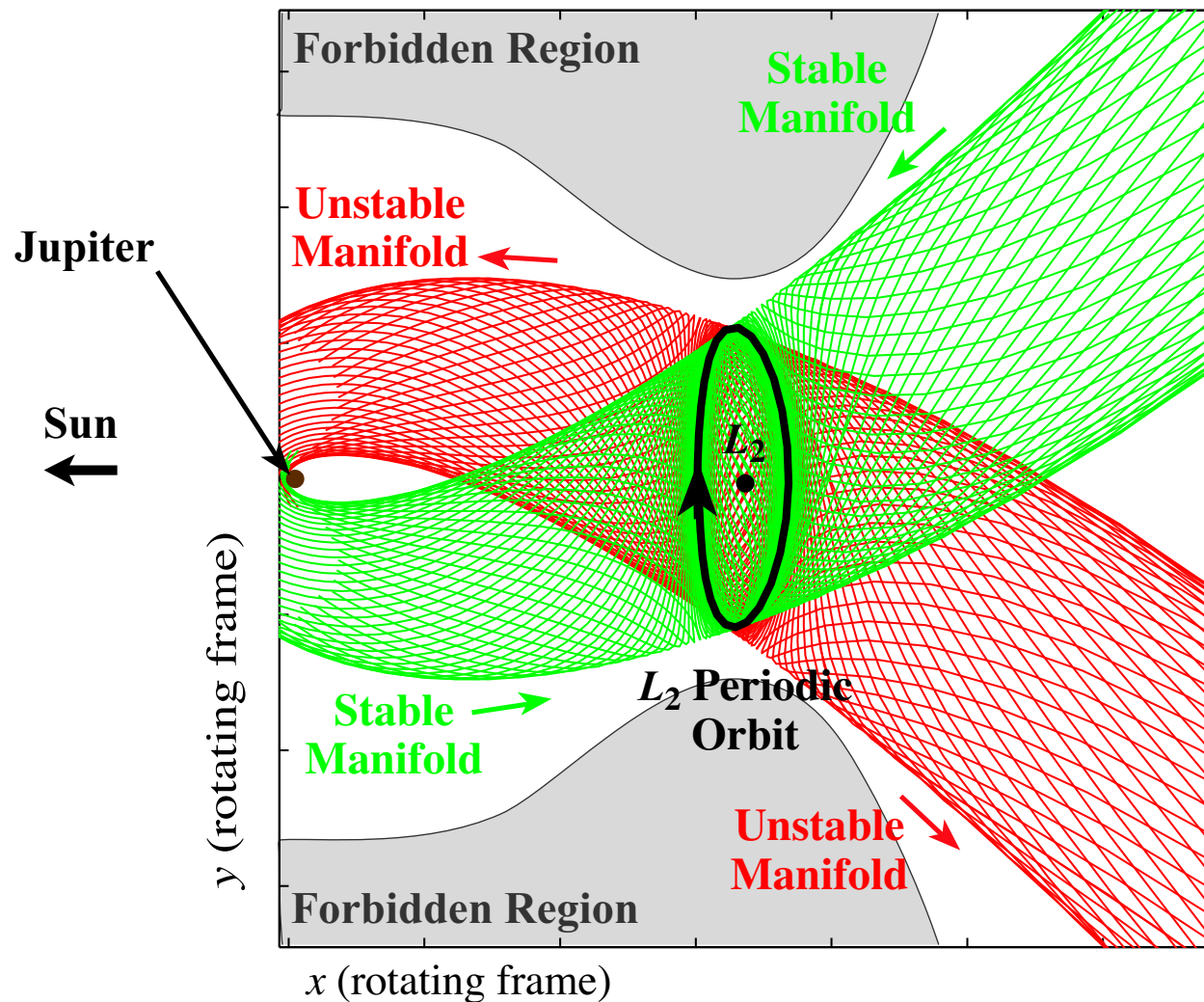
- For fixed energy, the equilibrium region $\simeq S^2 \times \mathbb{R}$.
 - Stable/unstable manifolds of periodic orbit define mappings between bounding spheres on either side of the barrier



Tubes in the 3-Body Problem

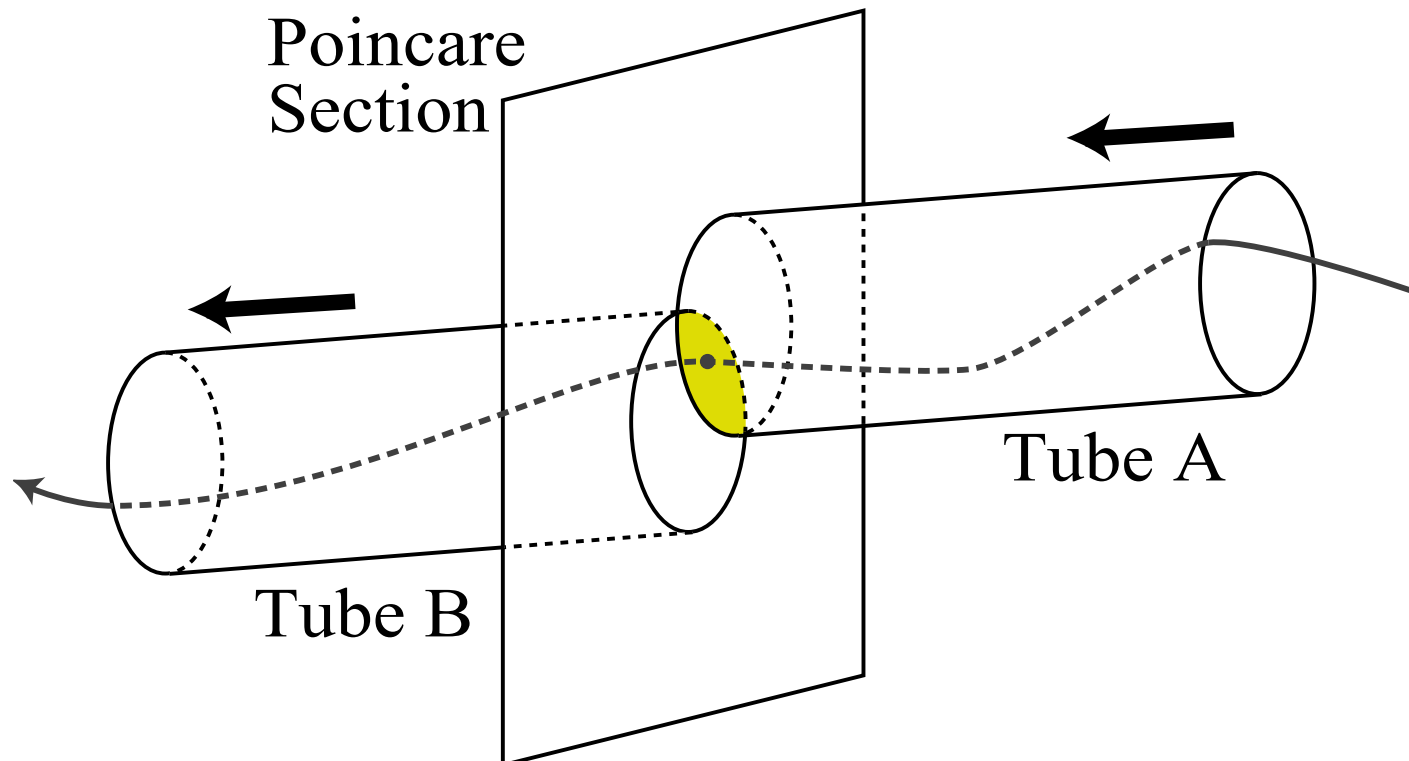
□ **Stable** and **unstable** manifold tubes

- Control transport through the potential barrier.



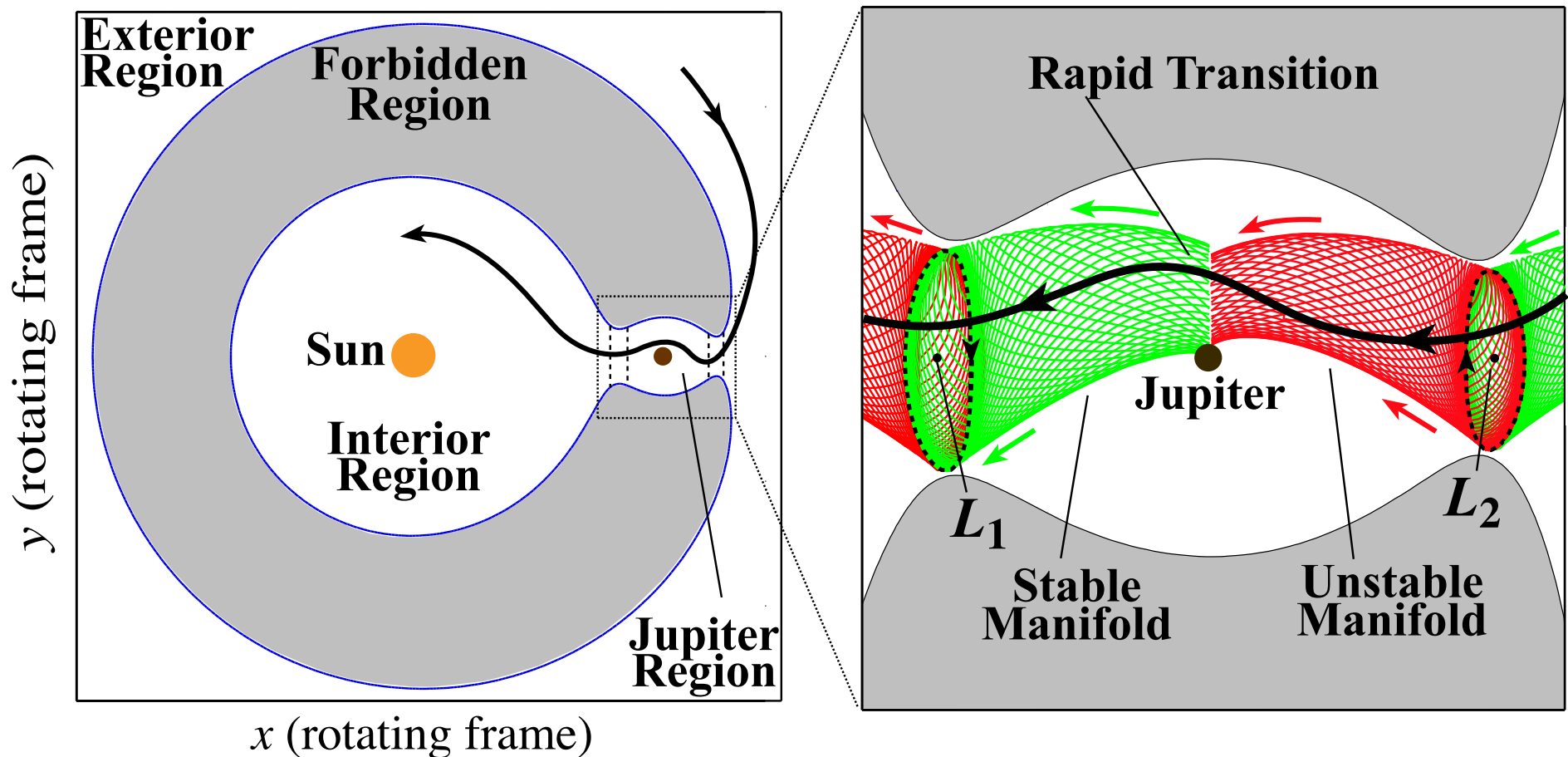
Transition Probabilities

- *Transport btwn non-adjacent regions*
 - Consider **intersections** between the interior of tubes — the transit orbits connecting regions.
 - Tube A and Tube B from different potential barriers.



Transition Probabilities

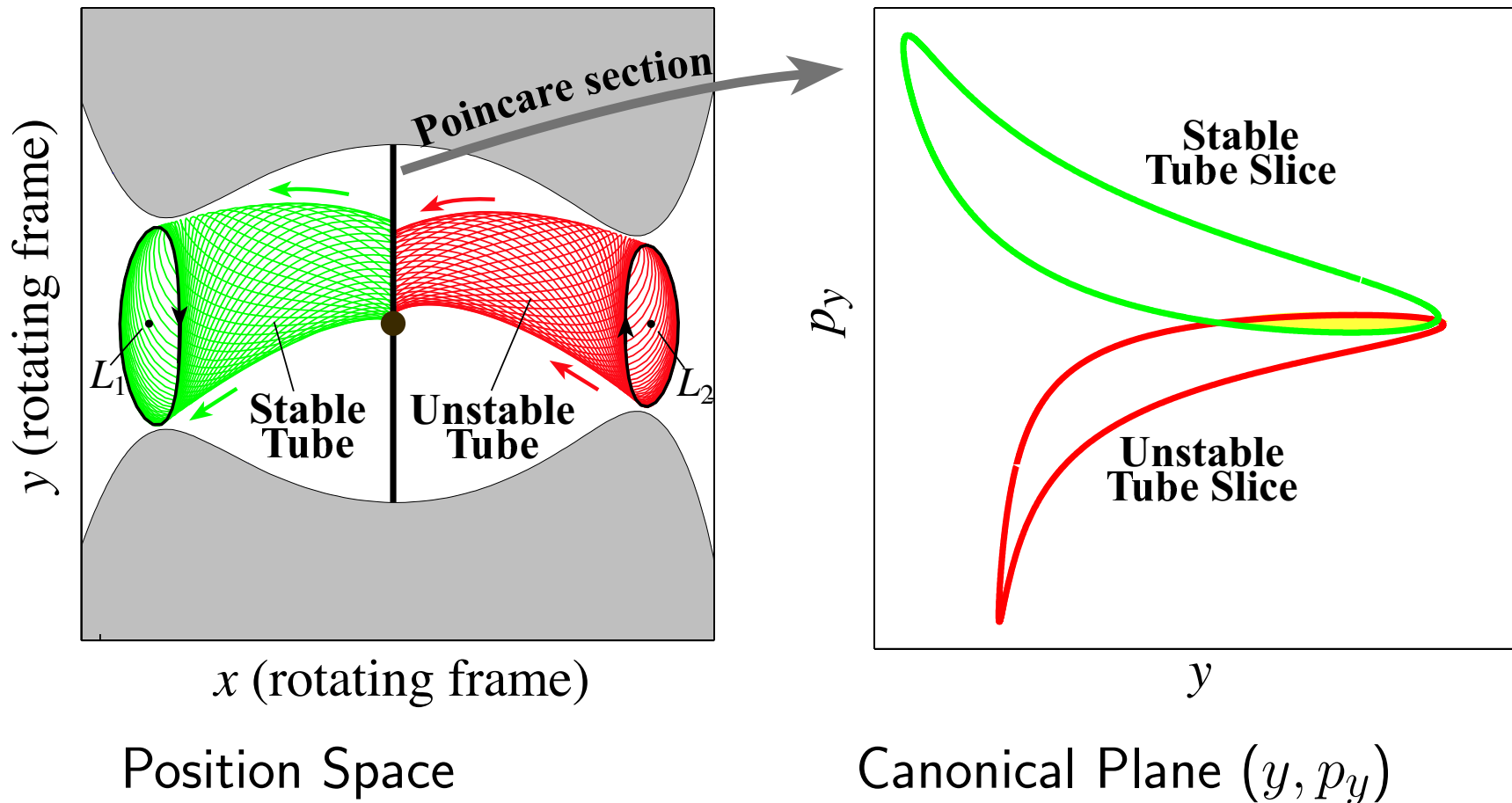
- Example: Comet transport between outside and inside of Jupiter (i.e., **Oterma**-like transitions)



(a)

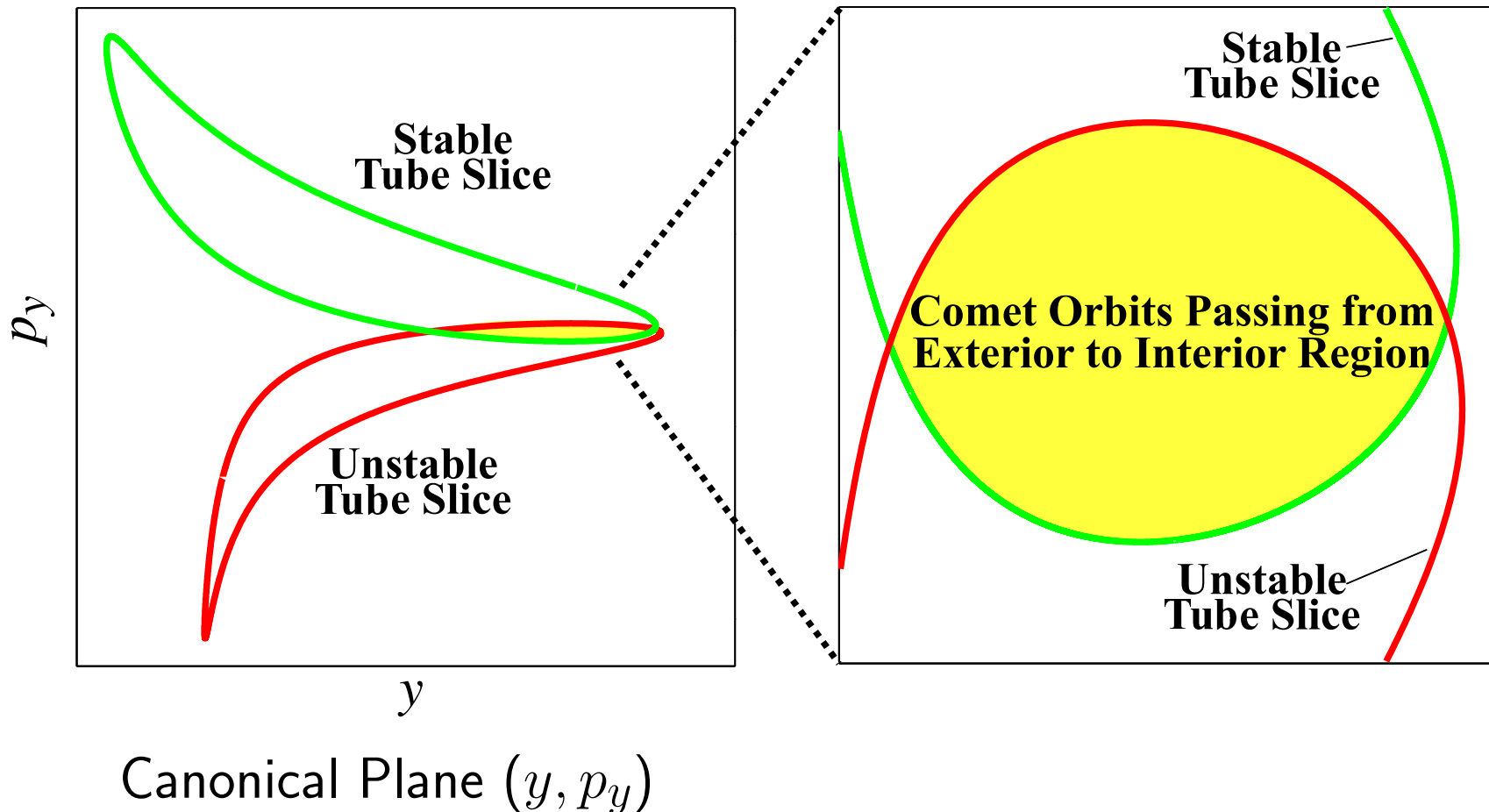
Transition Probabilities

- Consider Poincaré section intersected by both tubes.
- Choosing surface $\{x = \text{constant}; p_x < 0\}$, we look at the canonical plane (y, p_y) .



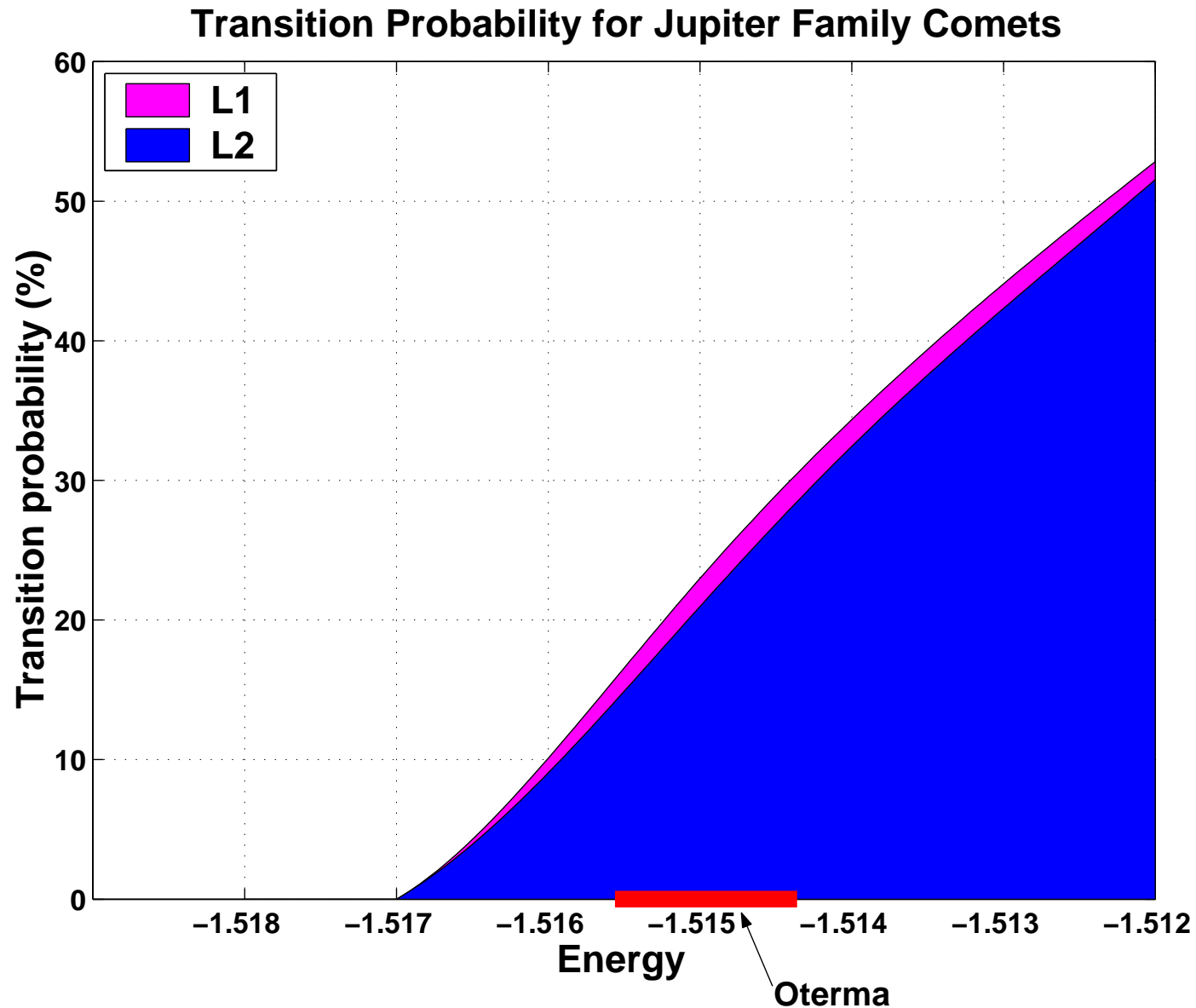
Transition Probabilities

- Canonical area ratio gives the **conditional probability** to pass from **outside** to **inside** Jupiter's orbit.
 - Assuming a well-mixed connected region on the energy mfd.



Transition Probabilities

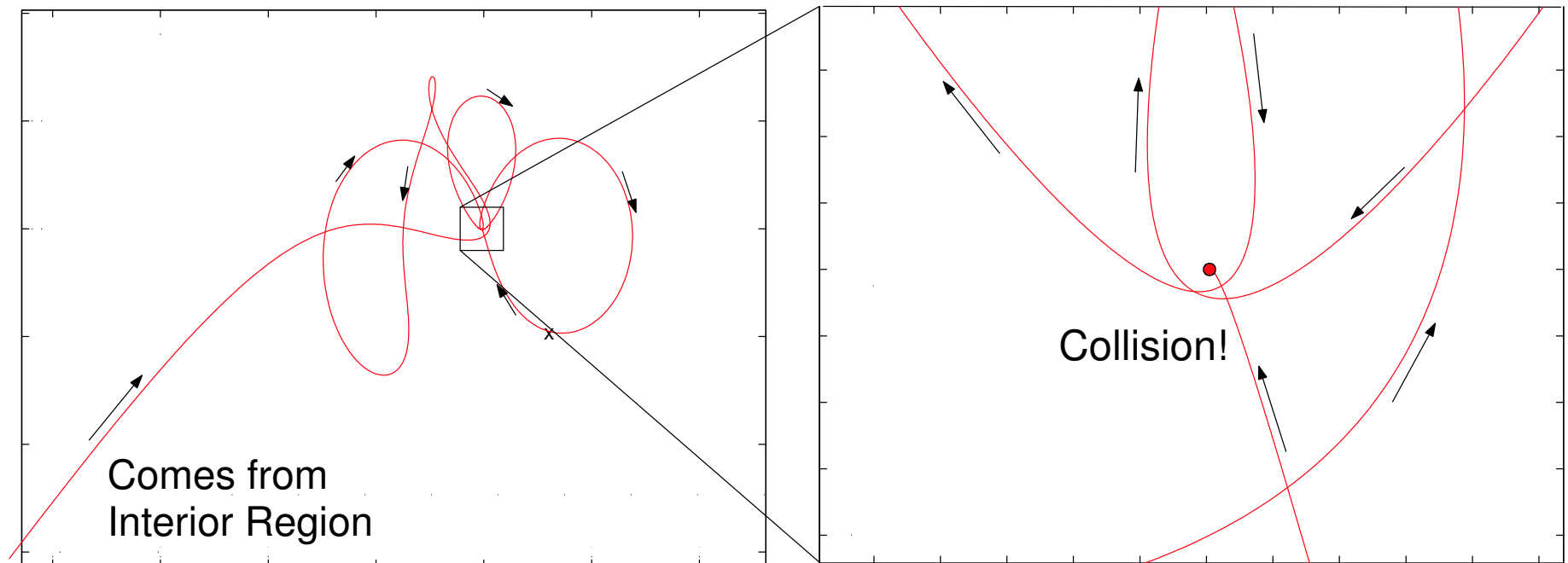
□ Jupiter family comet transitions: $X \rightarrow S$, $S \rightarrow X$



Collision Probabilities

- Low velocity impact probabilities
- Assume object enters the planetary region with an energy slightly above L1 or L2
 - eg, **Shoemaker-Levy 9** and **Earth-impacting asteroids**

Example Collision Trajectory



Collision Probabilities

■ *Collision probabilities*

- Compute from tube intersection with planet on Poincaré section
- Planetary diameter is a parameter, in addition to μ and energy E

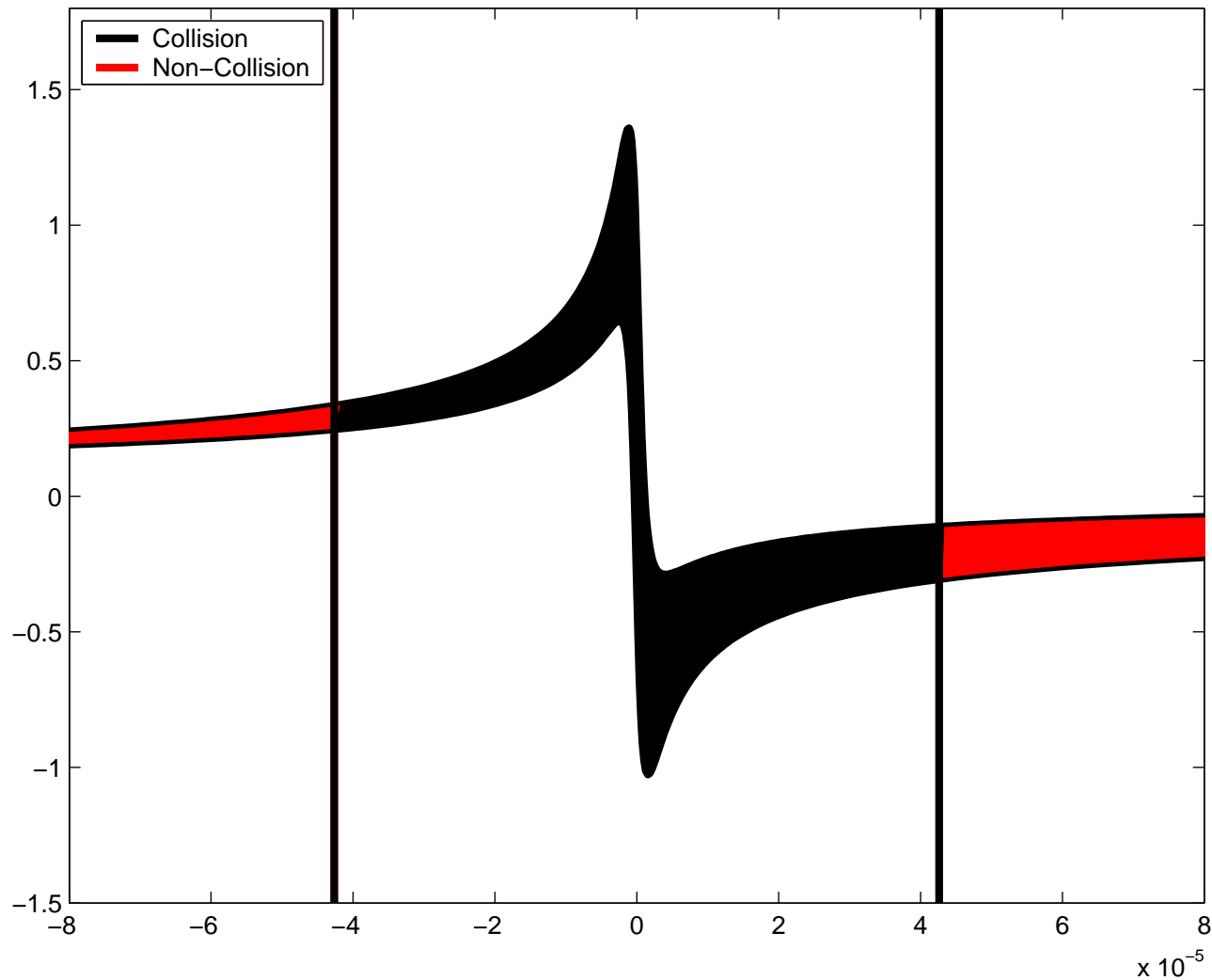


← Diameter of planet →

Collision Probabilities

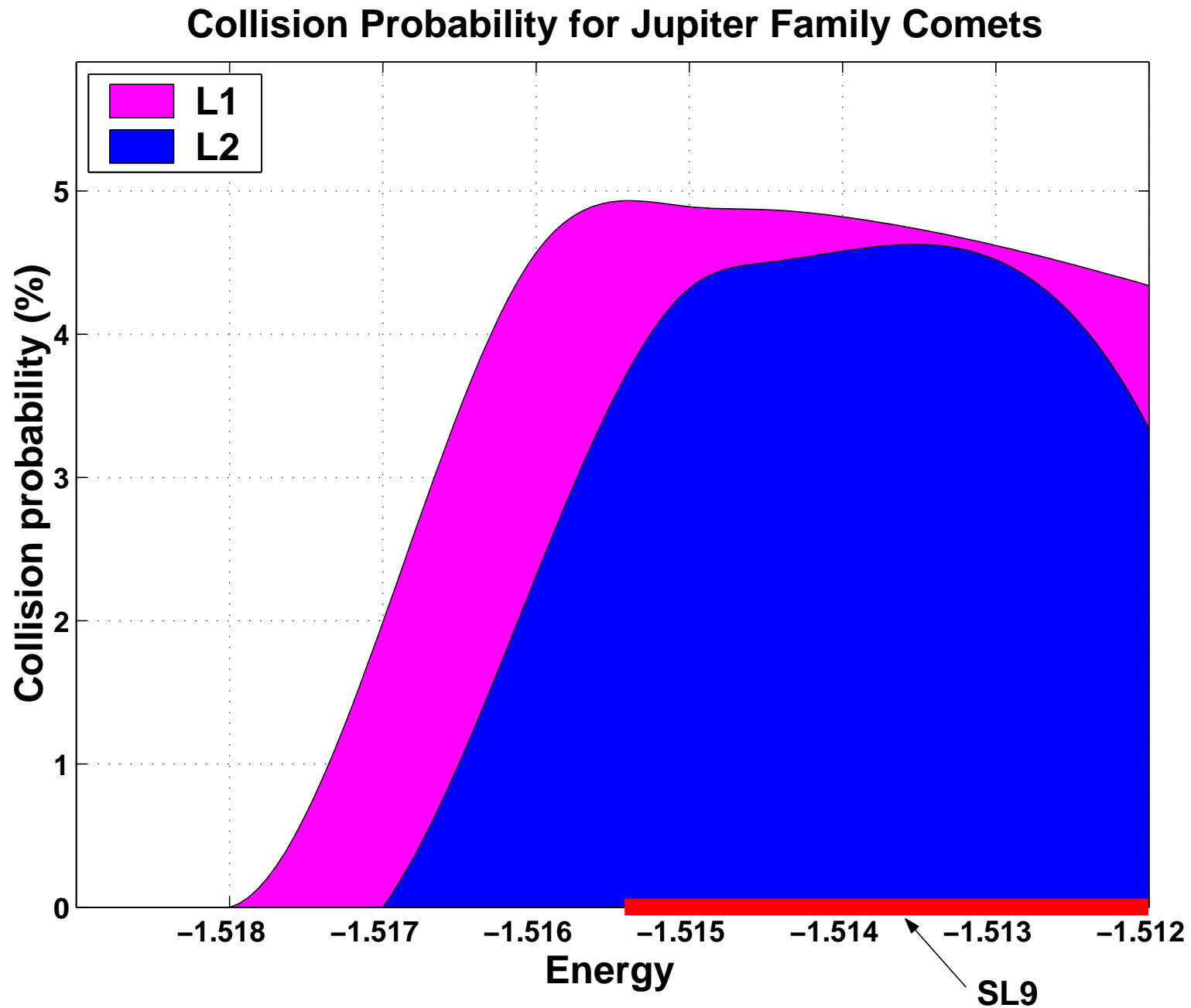
■ *Collision probabilities*

Poincare Section: Tube Intersecting a Planet

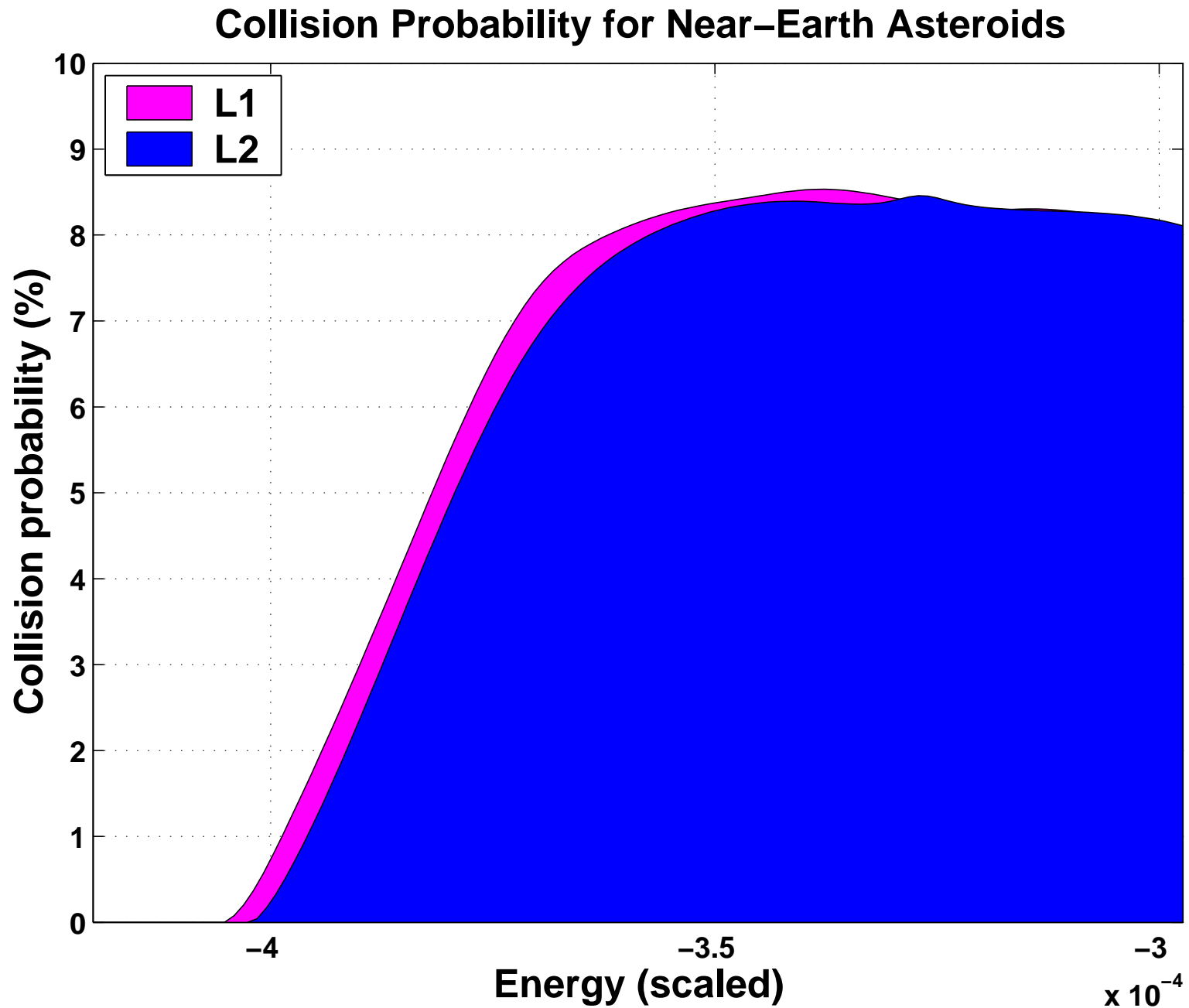


← Diameter of planet →

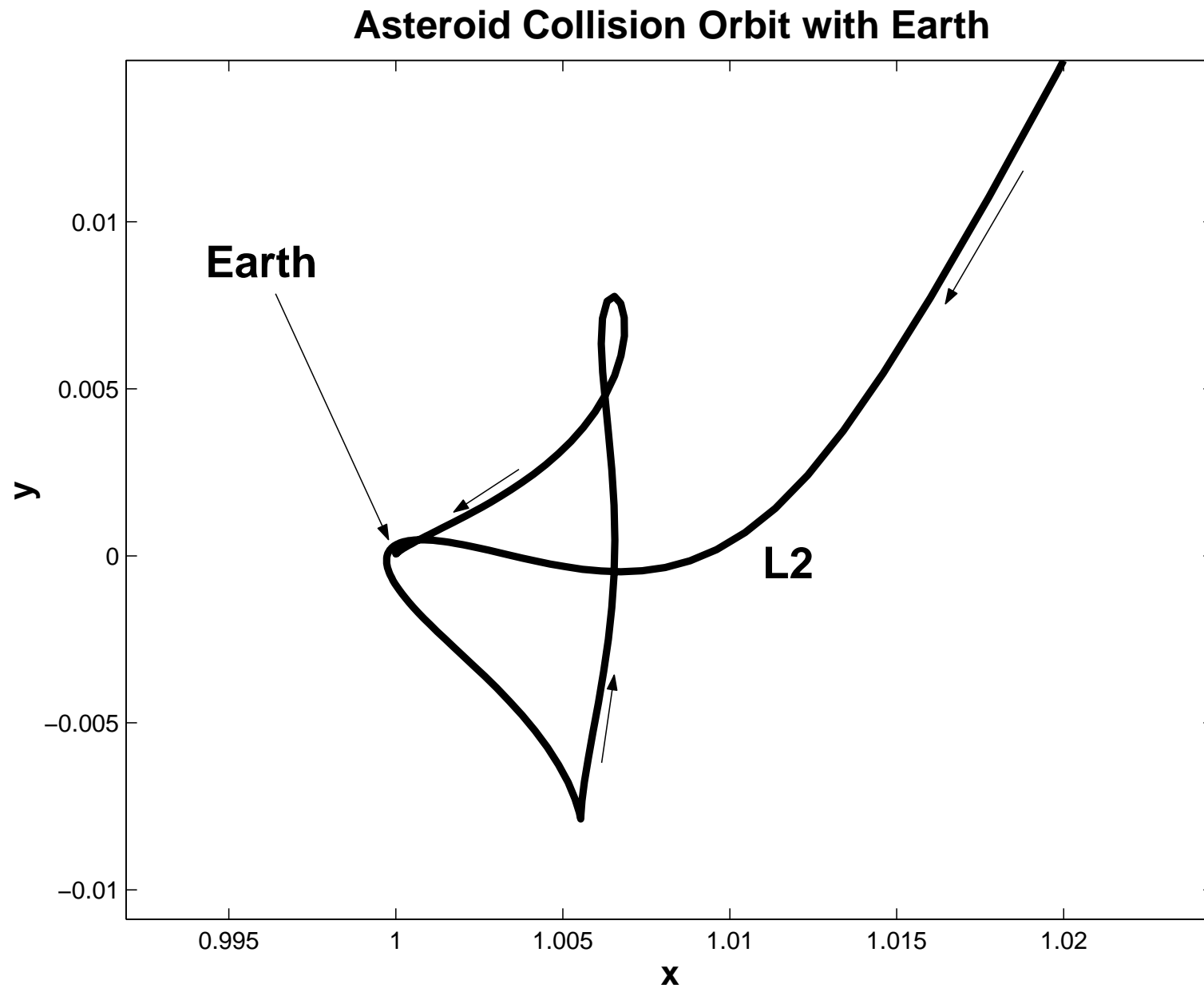
Collision Probabilities



Collision Probabilities



Collision Probabilities



Conclusion and Future Work

■ *Transport in the solar system*

- Approximate some solar system phenomena using the restricted 3-body problem
- Planar restricted 3-body problem
 - Stable and unstable manifold tubes of libration point orbits can be used to compute statistical quantities of interest
 - Probabilities of transition, collision
- Theory and observation agree

■ *Future studies to involve multiple three-body problems*

References

- Jaffé, C., S.D. Ross, M.W. Lo, J. Marsden, D. Farrelly, and T. Uzer [2002] *Statistical theory of asteroid escape rates*. *Physical Review Letters*, to appear.
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The End