Elastodynamic Analysis of a Prenotched Plate by the Meshless Local Petrov-Galerkin (MLPG) Method

H.K. Ching and R.C. Batra

Summary

We use the Meshless Local Petrov-Galerkin method to analyze transient deformations of a double edge prenotched plate with the smooth edge between the two notches loaded by uniformly distributed compressive tractions. The Newmark method is adopted for the time integration scheme. Stresses near the notch tip computed by the MLPG method agree with those obtained from the finite element solution. Time histories of the Mode-I and Mode-II stress intensity factors (DSIF) are determined from the computed stress fields.

Introduction

The meshless method has attracted a lot of attention in the past decade due to the flexibility of locating nodes. Atluri and Zhu [1] proposed a meshless method which requires no background mesh to integrate the weak formulation. Atluri et al. [2] have pointed out that the Galerkin approximation can also be adopted that leads to a symmetric stiffness matrix. Atluri and Zhu [3] solved elastostatic problems by the MLPG method, and found this method to be more accurate for computing stresses near holes than the FEM; the MLPG method has been used to solve a variety of boundary value problems. Lin and Atluri [4] introduced upwinding schemes in the present method to analyze the steady convection-diffusion problems. Ching and Batra [5] enriched the polynomial basis functions with those appropriate to describe singular deformation fields near a crack tip and used the diffraction criterion to find stress intensity factors, J-integrals and singular stress fields near a crack tip. Gu and Liu [6] used the Newmark family of methods to study forced vibrations of a beam. The problem of bending of a thin plate has been studied by Long and Atluri [7]. Warlock et al. [8] have analyzed elastostatic deformations of a material compressed in a rough rectangular cavity analytically by the Laplace transformation technique and numerically by the MLPG method.

Formulation of the Problem

Kalthoff and Winkler [9] in 1987 proposed an experiment to study transient mode-II deformations. It involves a double edge-notched plate with the edge between the two notches impacted by a fast moving cylindrical projectile. Here, we use the MLPG method to simulate this problem and approximate the action of the impactor by applying uniformly distributed compressive tractions on the impacted surface. Figure 1 shows a schematic sketch of the problem studied. We assume that a plane strain state of deformation prevails in the plate, and take Young’s modulus $E = 210\text{GPa}$, Poisson’s ratio $\nu = 0.29$, the mass density $\rho = 7833\text{kg/m}^3$, the radius of the circular notch tip $= 0.15\text{mm}$, the applied normal traction $\bar{t} = 200H(t)\text{MPa}$, where H(t) is the Heaviside step function, and tangential traction on the impacted surface $= 0$. Because of the symmetry of the problem about the horizontal centroidal

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1 Department of Engineering Science and Mechanics, MC 0219, Virginia Polytechnic Institute and State University, VA 24061, USA
plane, deformations of only the upper half of the plate are analyzed. Figure 2 exhibits the nonuniform nodal mesh of 3632 nodes with 25 nodes on the surface of the circular notch tip. The diffraction criterion [10] is used to account for the discontinuous deformation fields across the notch surface. Coupled ordinary differential equations obtained from the local symmetric weak formulation of the governing partial differential equations are integrated by the Newmark method with $\gamma = 1.5, \beta = 1$. The time integration scheme is unconditionally stable. However, we take time step $\Delta t = 0.0625\mu s$ in order to obtain accurate results. The two approximate solutions obtained by the MLPG and the FE methods are compared; the code ABAQUS was used to compute the FE solution. The same nodal mesh was used in the two analyses.

Figure 3 displays the undeformed and the deformed shapes of the notch tip. Due to the compressive tractions applied on the edge between the two notches, the notch faces move upwards. It is clear that the two sets of results agree well with each other. The time histories of normal stresses $\sigma_{11}$ and $\sigma_{22}$ at the notch tip are plotted in Fig. 4. The dilatational wave arrives at the notch tip at about $7\mu s$. Soon after the arrival of the wave, stresses at the notch tip increase with $\sigma_{22}$ being significantly larger in magnitude than $\sigma_{11}$. For time $t = 14\mu s$ and $24\mu s$, Figs. 5a and 5b evince, respectively, the variations of $\sigma_{22}$ and $\sigma_{12}$ at points directly ahead of the notch tip. We note that the axial variation of $|\sigma_{12}|$ exhibits a boundary layer like phenomenon near the notch tip; the thickness of the boundary layer equals 0.2% of the length of the notch. The angular distributions of the principal tensile stress and the maximum shear stress at $t = 14\mu s$ and $24\mu s$ are exhibited in Figs. 6a and 6b. The angular locations, $\theta$, of points where these stresses attain their maximum values are essential the same at $t = 14\mu s$ and $24\mu s$. Whereas the maximum principal tensile stress occurs at $\theta = 70^\circ$, the maximum shear stress attains its peak value at $\theta = -60^\circ$. These angular positions are close to those found by Batra and Gummalla [11] in the transient FE analysis of the thermoviscoelastic problem. The angular distributions of the hoop stress $\sigma_{00}$ and shear stress $\sigma_{10}$ are plotted in Fig. 7a and 7b respectively. Maximum values of $\sigma_{10}$ occur at the extremities of the circular surface of the notch tip.

We now determine the stress intensity factors based on the near-tip stress fields. For elastodynamic deformations of a body containing a stationary crack, the mode-I and the mode-II stress intensity factors can be determined from $K_I = \sqrt{2\pi r \sigma_{22}(t)}$ and $K_{II} = \sqrt{2\pi r \sigma_{12}(t)}$ respectively where $r$ is the distance straight ahead of the notch tip and should be taken within the singular-deformations dominated zone. At each time step we plotted, on a logarithmic scale, the variations of $|\sigma_{22}|$ and $|\sigma_{12}|$ with $r$. When plotting $|\sigma_{12}|$ vs. $r$, points outside of the boundary layer were taken. It is clear from the results plotted in Fig. 8a that indeed $K_I$ and $K_{II}$ are proportional to $\sqrt{r}$ during the time interval considered herein. The time histories of the stress intensity factors are depicted in Fig. 8b. Significantly larger values of $K_{II}$ relative to those of $K_I$ imply that the mode-II deformations near the notch tip are dominant.
Conclusions

We have used the MLPG method to study transient deformations of an elastic double edge prenotched plate with the smooth edge between the two notches loaded by uniformly distributed compressive tractions. The deformation and stress fields near the notch tip computed by the MLPG method essentially agree with those obtained from the finite element solution. It is found that deformations near the notch tip exhibit the $1/\sqrt{r}$ singularity. Therefore, the dynamic stress intensity factors can be computed from the near-tip fields.

References


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Fig. 1 Schematic sketch of the problem studied

Fig. 2 Nodal mesh for the upper half of the double edge-notched plate

Fig. 3 Undeformed and deformed shapes of the circular notch surface

Fig. 4 Time histories of $\sigma_{11}$ and $\sigma_{22}$ at the notch tip
**Fig. 5a** Variations of $\sigma_{22}$ with the distance directly ahead of the notch tip

**Fig. 5b** Variations of $\sigma_{12}$ with the distance directly ahead of the notch tip

**Fig. 6a** Angular distribution of the maximum principal stress on the notch surface at two different times

**Fig. 6b** Angular distribution of the maximum shear stress on the notch surface at two different times
Fig. 7a Angular distribution of the hoop stress on the notch surface at two different times

Fig. 7b Angular distribution of the shear stress on the notch surface at two different times

Fig. 8a The time history of the index of singularity for mode I and mode II deformations

Fig. 8b The time history of the stress intensity factors $K_I$ and $K_{II}$