Debonding of confined elastomeric layer using cohesive zone model

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Wavy or undulatory debonding is often observed when a confined/sandwiched elastomeric layer is pulled off from a stiff adherend. Here we analyze this debonding phenomenon using a cohesive zone model (CZM). Using stability analysis of linear equations governing plane strain quasi-static deformations of an elastomer, we find (i) a non-dimensional number relating the elastomer layer thickness, $h$, the long term Young’s modulus, $E_c$, of the interlayer material, the peak traction, $T_c$, in the CZM bilinear traction-separation (TS) relation, and the fracture energy, $\gamma_f$, of the interface between the adherend and the elastomer layer, and (ii) the critical value of this number that provides a necessary condition for undulations to occur during debonding. For the elastomer modeled as a linear viscoelastic material with the shear modulus given by a Prony series and a rate-independent bilinear TS relation in the CZM, the stability analysis predicts that a necessary condition for a wavy solution is that $T_c^2h/\gamma_fE_c$ exceeds 4.15. This is supported by numerically solving governing equations by the finite element method (FEM). Lastly, we use the FEM to study three-dimensional deformations of the peeling (induced by an edge displacement) of a flexible plate from a thin elastomeric layer perfectly bonded to a rigid substrate. These simulations predict progressive debonding with a fingerlike front for sufficiently confined interlayers when the TS parameters satisfy a constraint similar to that found from the stability analysis of the plane strain problem.

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1. Introduction

Systems consisting of a soft elastic or viscoelastic layer confined between two stiff substrates occur in numerous industrial applications. One example is manufacturing of bio-implants, which may involve mechanically demolding a soft polymer layer sandwiched between two relatively stiff molds [1]. A frequently observed phenomenon is the occurrence of contact undulations when a stiff layer is separated from the soft layer under tensile tractions. Classical examples include the formation of ripples when a contactor approaches an elastic film bonded to a fixed base [2–4], and wavy debonds in peel [5] and probe tack tests [6–8]. Experimentally, the characteristic spacing, $\lambda$, between two adjacent undulation peaks has been found [2,9] to scale linearly ($\lambda \approx 3–4h$) with the thickness $h$ of the confined interlayer while being independent of the interfacial adhesion properties. The linear stability analyses and energy arguments [10–13] have been used to show that undulations result from the competition between the strain energy of the system acting as a stabilizing influence, and the energy associated with the interfacial forces (such as van der Waals forces) acting as a destabilizing influence. These approaches give a threshold value of the interaction energy for the onset of instability. For example, it was shown [10,14] that the condition $\frac{\pi^2}{\lambda^2} \geq \frac{\gamma_f}{hE_c}$ is necessary for the onset of contact instabilities when a rigid contactor is gradually brought close to an elastic film of thickness $h$ and Young’s modulus $E$, where $A$ is the effective Hamaker constant for van der Waals interactions and $d$ the gap between the contactor and the film at the onset of instabilities. Other examples include [15] morphological changes in an elastic film caused by an applied electric field. Combined experimental and linear stability approaches have helped identify a threshold value of the effective voltage as a function of the film stiffness.

As debonding ensues at the interface between an adhesive and an adherend, multiple nonlinear processes such as cavitation and fibrillation may occur at the debonding site. These involve different length scales and contribute to the overall energy dissipation during the creation of the two new surfaces. In a cohesive zone model (CZM) [16,17], the collective influences of these small-scale mechanisms are lumped together into a traction-separation (TS) relation. In this approach the adjoining points on the two sides of an interface are conceived to be connected by a spring of zero length that begins softening with extension (separation) after reaching a critical extension and subsequently breaks upon
Nomenclature

Symbols

- **A**: Hamaker constant for van der Waals interaction
- **B**: Left Cauchy-Green tensor
- **d**: Distance between an elastic film and an approaching contactor
- **$\bar{d}$**: Damage variable
- **D**: Bending rigidity of the flexible plate
- **$E$**: Young’s modulus of the elastomeric interlayer
- **$h$**: Thickness of the elastomeric interlayer
- **$i = \sqrt{-1}$**: Imaginary unit
- **$K$**: Slope of the rising part of the straight line in the bilinear traction-separation law
- **$k$**: Wavenumber of the $x$-dependent part of the perturbation to the stream function and the hydrostatic pressure.
- **$m$**: Ratio of the modulus of the spring in the spring-dashpot link to the long-time modulus for the 1-term Prony series
- **$p$**: Hydrostatic pressure not related to strains for incompressible materials
- **$R$**: Reaction force on the rigid adherend
- **$T$**: Normal traction at the interface
- **$t$**: Time
- **$u$, $v$, $w$**: Displacement components along $x$, $y$ and $z$ directions, respectively
- **$x$, $y$, $z$**: Axes of the rectangular Cartesian coordinate system when the index $i$ of the system $x_i$ has values 1, 2 and 3, respectively
- **$\alpha$**: Magnitude of the slope of the falling part of the straight line portion of the bilinear traction-separation relation
- **$\Delta$**: Applied vertical displacement to the upper adherend
- **$\delta$**: Displacement jump at the interface, also called the contact opening
- **$\Delta_0$**: Dominant wavelength of debonding undulation
- **$\mu$**: Shear modulus of the elastomeric interlayer
- **$\psi$**: Stream function introduced to define displacement components $u$ and $w$
- **$\omega$**: Growth rate of a perturbation
- **$a_1$, $a_2$**: Material constants in the constitutive equation used to model finite strain viscoelasticity
- **$a_T$**: Shift factor relating the relaxation time at temperature $T$ to that at the reference temperature
- **$C_1$, $C_2$**: Constants in the Williams- Landel-Ferry (WLF) equation for $a_T$
- **$E_{01} = 3\mu_{00}$**: Long term Young’s modulus ($= 3 \times$ long-term shear modulus) of the viscoelastic material
- **$E_{00} = 3\mu_{00}$**: Instantaneous Young’s modulus ($= 3 \times$ instantaneous shear modulus) of the viscoelastic material
- **$F_t$**: Deformation gradient
- **$G$**: Fracture energy of the interface
- **$K_{softening} = T^2_{c} / G_c$**: Relaxation function normalized by the instantaneous modulus of the viscoelastic material
- **$K_{elastic} = E_{\infty} / h$**: Components of the infinitesimal strain tensor
- **$l_{finger}$**: Length of a finger
- **$p_{inh}$**: Non-homogeneous perturbation to the hydrostatic pressure
- **$T_c$**: Peak value of the normal traction at the interface
- **$\chi_i$**: Axes of the rectangular Cartesian coordinate system
- **$\delta_c$**: Critical displacement jump when damage initiates
- **$\delta_1$**: Displacement jump at the initiation of debonding/separation
- **$\epsilon_{ij}$**: Components of the stress tensor
- **$\mu_i$**: Shear modulus of the $i$th term in the generalized Maxwell model used to define the relaxation function of the viscoelastic model
- **$\mu_R$**: Relaxation function for the elastomeric interlayer when modeled as a linear viscoelastic material
- **$\sigma_{ij}$**: Components of the stress tensor
- **$\tau_1$**: Characteristic relaxation time of the $i$th element of the generalized Maxwell model
- **$\phi_{c1}$**: The lower limit of $\phi$ for debonding instability
- **$\phi_{c2}$**: The value of $\phi$ beyond which region III sets in
- **$\psi_{inh}$**: Non-homogeneous perturbation of the stream function
- **$A$**: Area of the interlayer initially bonded to the rigid adherend
- **$\rho$**: $z$-dependent part of the hydrostatic pressure perturbation
- **$\bar{T}$**: Temperature
- **$T_{REF}$**: Reference temperature
- **$\frac{\partial}{\partial x_i}$**: Partial derivative (with respect to $x_i$) operator
- **$\bar{x}$**: Distance along the $x$ axis normalized by the half width of the rigid adherend
- **$\bar{\Delta}$**: Rate of the applied vertical displacement to the upper adherend
- **$\bar{\delta}$**: Contact opening normalized by $\delta_1$
- **$\bar{\psi}$**: $z$-dependent part of the stream function perturbation
- **$\bar{\omega}$**: The growth rate $\omega$ normalized by $1/\tau_1$
reaching a larger limiting value of the extension. In a typical TS relation, it is assumed that the energy associated with the softening process is irreversible, i.e., upon unloading from extension between the critical and the limiting values, the spring stiffness remains constant at the reduced value. At the limiting extension value, the area under the TS curve equals the critical fracture energy ($G_c$) of the interface whose value is generally derived from the test data. Works cited above addressing the interplay between the destabilizing contact interaction, and the stabilizing elastic deformations of a film and the concomitant debonding instability suggest that the threshold for undulatory separation can be modeled by a TS relation. Hui et al. [18] illustrated this for debonding between two blocks made of the same material.

In order to understand the mechanics of the peeling and demolding processes, researchers have often relied on numerical simulations. Since the use of the CZM coupled with the finite element method (FEM) by Hillerborg et al. [19] to study fracture problems, significant progress has been made in modeling interfacial debonding/delamination of a polymer interlayer [20–23]. However, there has been limited research [8,24,25] on capturing progressive interfacial debonding undulations using the CZM. The formation of debonding undulations (such as fingering in a peeling problem) adds to the complexity of the mechanics of the demolding process. The development of a tool for capturing such phenomena is important for delineating the debonding process and improving our understanding of the associated mechanics. The focus of our work is to identify the role of the TS, the material, and the geometric parameters in causing undulatory debonding.

The rest of the paper is organized as follows: In Section 2, we present a linear stability analysis of plane strain deformations of a semi-infinite elastomeric layer debonding from a rigid adherend pulled vertically outward. We find a non-dimensional parameter in terms of the TS and the elastic film parameters that must exceed a critical value for deboning to exhibit an undulatory morphology. In Section 3, we analyze the two-dimensional (2D) problem by the FEM to provide details of the debonding evolution and confirm the necessary condition derived by the linear stability analysis. In Section 4, we use the FEM to analyze a practical problem, namely 3D deformations when a flexible plate is peeled off a confined elastomeric layer bonded to a rigid substrate. This necessitates introducing another non-dimensional parameter in terms of the plate bending stiffness and elastomer properties. We correlate results of this problem with our learnings from Sections 2 and 3. Conclusions are summarized in Section 5.

2. Analytical approach

2.1. Problem formulation

We investigate the initiation of wavy debonding using a CZM when a rigid smooth adherend bonded to the upper surface of an elastomeric layer is pulled upwards while its lower surface stays perfectly bonded to the rigid support. Our analysis is essentially similar to those of Shenoy and Sharma [10] and Huang et al. [14] who used a linear stability approach to study instabilities in an elastic layer triggered by van der Waals forces when a rigid probe is brought near the layer. However, we use a TS relation to simulate debonding between the elastomeric layer and the upper rigid adherend. The configuration studied is schematically shown in Fig. 1. The infinitely strong perfect bonding between the lower stationary rigid adherend and the elastomer layer is modeled by constraining to zero displacements of points on the bottom surface of the interlayer. We assume that the interlayer material is isotropic, incompressible, homogeneous, and linear elastic/viscoelastic. The problem is first analyzed for a linear elastic interlayer, and subsequently for a viscoelastic layer. For the assumed form of the solution using the separation of variables with evolution in time represented by a sine (or cosine) function, the constitutive relation for a linear viscoelastic material reduces to that of a linear elastic material with the shear modulus depending upon a solution variable.

2.1.1. Linear elastic material

The constitutive equation for an incompressible, homogeneous and isotropic linear elastic material is $\sigma_{ij} = -p\delta_{ij} + 2\mu \varepsilon_{ij}$, where $\sigma_{ij}$ is the stress tensor (Each of the indices $i, j$ and $k$ corresponds to $x, y$, and $z$ directions), $\varepsilon_{ij}$ the strain tensor for infinitesimal deformations, $p$ the hydrostatic pressure not related to strains, and $\mu$ the shear modulus of the interlayer material. Recalling the strain–displacement relation, $\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$, the incompressibility constraint, $\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = 0$, and assuming zero body force and negligible inertial effects the equilibrium equations $\frac{\partial \sigma_{ii}}{\partial x_i} = 0$ (repeated index $j$ is summed over $x, y$, and $z$) reduce to the following Navier’s equations.

$$\frac{\partial p}{\partial x} = \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

Here and below we denote the displacement components $u_x$ and $u_y$ as $u$ and $w$, respectively. Perfect bond with the fixed lower rigid adherend is incorporated by setting $u$ and $w$ equal to zero at the lower interface of the interlayer. For thin confined infinitely wide incompressible films under tension, the shear stress over the central region is negligible, e.g., see Lindsey et al. [26]. Thus, boundary conditions are:

Fixed base : $u(x, -h) = w(x, -h) = 0$

Surface tractions : $\begin{cases} \sigma_{zz}(x, 0) = -p(x, 0) + 2\mu \frac{\partial w}{\partial x}(x, 0) = T(\delta) \\ \sigma_{zz}(x, 0) = \mu \left( \frac{\partial w}{\partial x}(x, 0) + \frac{\partial u}{\partial z}(x, 0) \right) = 0 \end{cases}$

(2)

The interfacial normal traction $T$ is related to the displacement jump $\delta = \Delta - w(x, 0)$ by the TS relation that characterizes the interface between the interlayer and the upper adherend. Here $\Delta$ is the vertical displacement of the upper rigid adherend. We have tacitly assumed that the region of interest is far removed from the edges $x = \pm \frac{L}{2}$ and boundary conditions at these edges, not specified in Eq. (2), do not affect deformations in the interior.

2.1.1.1. Description of cohesive zone behavior at the interface. We consider a bilinear TS relation [27] for the interface, illustrated in Fig. 2, and given by Eq. (3). That is, the traction first increases linearly with the displacement jump, commonly termed the contact opening/separation, over the region $OA$. Point A denotes
initiation of damage/softening beyond which the traction decreases affinely with an increase in the contact opening (line AB). Should unloading occur at point M, the traction follows the path MO. Subsequent reloading occurs along the path OMB.

\[ T(\delta) = \begin{cases} \frac{K\delta}{\delta_B - \delta_C} & (0 \leq \delta \leq \delta_C) \\ \frac{T_C}{\delta_B - \delta_C} (\delta_B - \delta) & (\delta_C \leq \delta \leq \delta_B) \\ 0 & (\delta_B \leq \delta) \end{cases} \]  

(3)

Point B signifies complete separation at a point on the interface. The energy release rate at debonding (i.e., the fracture energy) equals the area of the triangle OAB. That is,

\[ \psi_c = \frac{1}{2} T_c \delta_f \]  

(4)

A complete description of the bilinear TS relation involves specifying the initial slope \( K \), the peak traction \( T_c \), and the fracture energy \( \psi_c \). In general, the value of \( K \) must be large enough to not significantly alter the effective stiffness of the system and prevent interpenetration under compression. A very large value of \( K \) can make the system matrices ill-conditioned when the problem is analyzed by the FEM. While \( \psi_c \) can be obtained directly from test data [28,29], finding values of parameters, \( T_c \) and \( \delta_f \), is difficult. An indirect method, frequently employed [22,30], is to iteratively find values of these parameters which, when used in numerical simulations, can predict reasonably well the experimental load-displacement traces. Recently, the digital image correlation [31] and the interferometry techniques [32] have been used to measure the TS parameters.

2.1.1.2. Homogeneous solution. We note that the homogeneous solution \((u = 0, w = 0, p = \text{constant})\) satisfies equilibrium Eq. (1) and boundary conditions (2) for \( T(\delta) = -p \). The corresponding stresses are \( \sigma_{zz} = \sigma_{zz} = \sigma_{yy} = -p \) and \( \sigma_{xz} = 0 \). The pressure \( p \), found from Eqs. (2a) and (3a), is given by Eq. (5). This homogeneous solution implies that pulling the upper adherend upwards will stretch the fictitious CZ springs while the interlayer will not deform.

\[ p = \begin{cases} -K\Delta & (0 \leq \Delta \leq \delta_c) \\ -\frac{T_c}{\delta_B - \delta_C} (\delta - \delta) & (\delta_c \leq \Delta \leq \delta_B) \\ 0 & (\delta_B \leq \Delta) \end{cases} \]  

(5)

2.1.1.3. Non-homogeneous solution. We now explore the possibility of a non-homogeneous solution of the boundary value problem defined by Eqs. (1–3). For the deformation to identically satisfy the incompressibility constraint, we write displacements in terms of a stream function \( \psi \) as

\[ u = \frac{\partial \psi}{\partial z}, \quad w = -\frac{\partial \psi}{\partial x} \]  

(6)

where \( \psi \) is a twice continuously differentiable function of \( x \) and \( z \). In order to find necessary conditions for the instability of the homogeneous solution \( \psi = \text{constant} \) and \( p = \text{constant} \) (defined by Eq. (5)), we perturb it by adding to it the non-homogeneous field: \( \psi^{nh}(x,z) = e^{ikx} \hat{\psi}(z) \) and \( \psi^{nh}(x,z) = e^{ikx} \hat{\psi}(z) \). Here the superscript \( nh \) stands for non-homogeneous, \( i = \sqrt{-1} \), and \( k \) represents the wavenumber of the perturbation.\(^1\) The perturbed form, when substituted into equilibrium Eq. (1), yields the following system of ordinary differential equations (ODEs).

\[ \mu \left( \frac{d^2 \hat{\psi}}{dz^2} - k^2 \hat{\psi} \right) - k \hat{\psi} = 0 \]  

(7)

The ODEs (7) have the solution

\[ \hat{\psi}(z) = \cosh(kz)A_1 + ik^2 \mu \sinh(kz)A_2 - i\mu \sinh(kz)A_3 \]  

\[ \hat{\psi}(z) = \frac{[i(\delta_B \Delta - \delta_c k^2 \mu \sinh(kz) + 2k^2 \mu \sinh(kz)A_2 + 2k^2 \mu \sinh(kz)A_3)]}{2k^2 \mu} \]  

(8)

where constants \( A_1, \ldots, A_4 \) of integration are determined from the boundary conditions. Recalling that \( \Delta \) is not perturbed, \( \delta = \Delta - w(x,0) = \Delta + ik\delta \hat{\psi}(0) \). In terms of the non-homogeneous \( z \)-dependent terms, boundary conditions (2) for \( \hat{\psi}(z) \) take the form:

\[ \begin{align*} 
\text{Fixed base:} & \quad \hat{\psi}(h) = \frac{d\hat{\psi}}{dz}(h) = 0 \\
\text{Top face:} & \quad \begin{cases} \hat{\psi}(0) + 2\mu k^2 \frac{d\hat{\psi}}{dz}(0) + ak\hat{\psi}(0) = 0 \\
\frac{d^2 \hat{\psi}}{dz^2}(0) + k^2 \hat{\psi}(0) = 0 \end{cases} 
\end{align*} \]  

(9a, b, c, d)

where

\[ \alpha = \begin{cases} -\frac{K}{\mu} & (0 \leq \delta \leq \delta_c) \\
\frac{\delta_B - \delta_c}{\delta_B - \delta} & (\delta_c \leq \delta \leq \delta_B) \\
0 & (\delta_B \leq \delta) \end{cases} \]  

(10)

We assume that \( \delta \) is increased monotonically during the debonding process. Two out of four constants in solution (8) are evaluated from boundary conditions (9a, b, c, d). Enforcing the remaining boundary conditions (9c, d) we obtain the eigenvalue problem \([C][A] = 0\) that has a non-trivial solution if and only if \( \det(C) = 0 \). This gives the following equation for \( k \) of which we seek real roots.

\[ f(hk, \alpha) = ah - \mu = 0 \]  

(11)

where \( f(hk) = \frac{2\beta(1 + \gamma^2)^2 \beta x^2 + 2\beta(1 + \gamma^2)\beta x^2 \mu}{2\beta(1 + \gamma^2)\beta x^2 \mu} - 1. \) Roots of Eq. (11) depend upon values of the TS parameters through their dependence upon \( \alpha \), the interlayer thickness \( h \), and the shear modulus \( \mu \), of the interlayer material. The function \( f \) versus \( hk \), plotted in Fig. 3, has the minimum value 6.22. Thus Eq. (11) has real roots only if \( ah/\mu \geq 6.22 \), and \( \alpha \) is positive. Eq. (10) implies that the separation, \( \delta \), must have values greater than \( \delta_c \) and at most equal to \( \delta_B \).\(^1\) The assumed perturbation, in the form of separation of variables with the complex \( x \)-dependent term, implies a sinuosoidal variation in \( x \)-direction of wave-number \( k \). Any perturbation can be considered for finding necessary conditions. However, finding sufficient conditions is more challenging.
For $\frac{a h}{\mu} > 6.22$, one real root of Eq. (11) is less than 2.12 and the other greater than 2.12. The perturbed solution can be expressed as a linear combination of these two solutions. It is possible that the two perturbations will grow at different rates.

2.1.2. Linear viscoelastic material

For a linear viscoelastic interlayer we assume the following constitutive relation.

$$\sigma_\zeta(x, z, t) = -\epsilon(x, z, t)\delta_\zeta + \int_0^T 2\mu_\tau \left( \frac{T - S}{\tau_\zeta} \right) \frac{d\epsilon_\zeta(x, z, S)}{dS} ds$$  \hspace{1cm} (12)

Here $\epsilon_\zeta(x, z, t) = 0$ for $t \leq 0$, $t$ is the present value of time and $\mu_\tau$ is the relaxation function expressed as Prony series [33]. $\mu_\tau(t) = \mu_\tau^i + \int_\tau^T \mu_\tau e^{-\frac{t - \tau_\zeta}{\tau_\zeta}} d\tau$, where $\mu_\tau^i$ and $\tau_\zeta$ denote, respectively, the shear modulus and the relaxation time. For discussion later, the dependence of $\tau_\zeta$ upon the temperature $T$ is assumed to be uniform in the interlayer material. Either by setting $aT/\mu_\zeta = 0$ or from Fig. 3, we get $hk = 0.212$, $f(hk) = 6.22$. We call values of variables for $k = 2/12$ critical and denote them by a subscript $c$. Thus $\lambda_c = (2\pi/k) = 2.96$, and for this value of $hk$, $f(hk) > 0$. Between two critical values, $\phi_{1c} \approx 6.22$ and $\phi_{2c} \approx 6.22(1 + m)$, the growth rate $\lambda$ of perturbations is positive for all wavenumbers lying between roots of $f(hk) = \phi$, as shown in Fig. 4a in which we have plotted the growth rate as a function of the wavenumber for values of $\phi$ in the range $[\phi_{1c}, \phi_{2c}]$.

Thus the interaction parameter $\phi$ must exceed the threshold $\phi_{1c}$, for a spatially sinusoidal perturbation to grow. Beyond the upper critical value, $\phi_{2c}$, of $\phi$ the growth rate becomes negative for wavenumbers lying between the two roots of $f(hk) = \phi$, as illustrated in Fig. 4b in which we have plotted the growth rate as a function of wavenumber for a $\phi > \phi_{2c}$. Contours in the wave number-\phi plane of the normalized growth rate $\lambda$ of perturbations for $m = 3$ and $\alpha_t = 1$ are shown in Fig. 5. For values of $\phi$ in region I ($\phi < \phi_{1c}$), perturbations will not grow and the adherend will snap off the interlayer when the debonding criterion is satisfied. In region II ($\phi_{1c} < \phi < \phi_{2c}$), perturbations will grow with growth rate depending upon the wavenumber $k$. For a given value of $\phi$ the undulation will consist of infinitely many wavenumbers with positive growth rates, and the resulting displacement will not be a pure sine curve. In order to delineate this, we need to find amplitudes of perturbations, which is beyond the scope of the linear analysis. The numerical solution of the problem by the FEM reported in Section 3 provides details of the debond nucleation and evolution. It is possible that the wavelength with the maximum growth rate will determine the minimum spacing between adjacent undulations as stipulated by Wright and Ockendon [34] in their study of adiabatic shear bands.

It can be noted from Fig. 4a that the range of wavenumbers with positive growth rate increases as a function of $\phi$, with the wavenumber $2.12/h$ having the fastest growth rate that is independent of $\phi$. The growth rate as a function of the wavenumber plotted in Fig. 4b illustrates an example of the behavior when $\phi > \phi_{2c}$, i.e., region labeled III in Fig. 5. While the wavelength $\lambda_c = 2.96h$ is no longer expected in this region, two wavelengths with

$$f < \phi < (m + 1)f$$  \hspace{1cm} (15)

Eq. (14) and inequalities (15) suggest that the fastest growing wavelength corresponds to the minimum value of $f(hk)$ which is independent of values assigned to the CZM parameters and the interlayer material. Either by setting $\frac{aT}{\mu_\zeta} = 0$ or from Fig. 3, we get $hk = 2.12$, $f(hk) = 6.22$. We call values of variables for $k = 2/12$ critical and denote them by a subscript $c$. Thus $\lambda_c = (2\pi/k) = 2.96$, and for this value of $hk$, $f(hk) > 0$. Between two critical values, $\phi_{1c} \approx 6.22$ and $\phi_{2c} \approx 6.22(1 + m)$, the growth rate $\lambda$ of perturbations is positive for all wavenumbers lying between roots of $f(hk) = \phi$, as shown in Fig. 4a in which we have plotted the growth rate as a function of the wavenumber for values of $\phi$ in the range $[\phi_{1c}, \phi_{2c}]$.

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$$f < \phi < (m + 1)f$$  \hspace{1cm} (15)

Eq. (14) and inequalities (15) suggest that the fastest growing wavelength corresponds to the minimum value of $f(hk)$ which is independent of values assigned to the CZM parameters and the interlayer material. Either by setting $\frac{aT}{\mu_\zeta} = 0$ or from Fig. 3, we get $hk = 2.12$, $f(hk) = 6.22$. We call values of variables for $k = 2/12$ critical and denote them by a subscript $c$. Thus $\lambda_c = (2\pi/k) = 2.96$, and for this value of $hk$, $f(hk) > 0$. Between two critical values, $\phi_{1c} \approx 6.22$ and $\phi_{2c} \approx 6.22(1 + m)$, the growth rate $\lambda$ of perturbations is positive for all wavenumbers lying between roots of $f(hk) = \phi$, as shown in Fig. 4a in which we have plotted the growth rate as a function of the wavenumber for values of $\phi$ in the range $[\phi_{1c}, \phi_{2c}]$.

Thus the interaction parameter $\phi$ must exceed the threshold $\phi_{1c}$, for a spatially sinusoidal perturbation to grow. Beyond the upper critical value, $\phi_{2c}$, of $\phi$ the growth rate becomes negative for wavenumbers lying between the two roots of $f(hk) = \phi$, as illustrated in Fig. 4b in which we have plotted the growth rate as a function of wavenumber for a $\phi > \phi_{2c}$. Contours in the wave number-\phi plane of the normalized growth rate $\lambda$ of perturbations for $m = 3$ and $\alpha_t = 1$ are shown in Fig. 5. For values of $\phi$ in region I ($\phi < \phi_{1c}$), perturbations will not grow and the adherend will snap off the interlayer when the debonding criterion is satisfied. In region II ($\phi_{1c} < \phi < \phi_{2c}$), perturbations will grow with growth rate depending upon the wavenumber $k$. For a given value of $\phi$ the undulation will consist of infinitely many wavenumbers with positive growth rates, and the resulting displacement will not be a pure sine curve. In order to delineate this, we need to find amplitudes of perturbations, which is beyond the scope of the linear analysis. The numerical solution of the problem by the FEM reported in Section 3 provides details of the debond nucleation and evolution. It is possible that the wavelength with the maximum growth rate will determine the minimum spacing between adjacent undulations as stipulated by Wright and Ockendon [34] in their study of adiabatic shear bands.

It can be noted from Fig. 4a that the range of wavenumbers with positive growth rate increases as a function of $\phi$, with the wavenumber $2.12/h$ having the fastest growth rate that is independent of $\phi$. The growth rate as a function of the wavenumber plotted in Fig. 4b illustrates an example of the behavior when $\phi > \phi_{2c}$, i.e., region labeled III in Fig. 5. While the wavelength $\lambda_c = 2.96h$ is no longer expected in this region, two wavelengths with
infinite growth rate will be close to \( \lambda_\text{c} = \frac{2\pi h}{\eta_1} \) and \( \frac{2\pi h}{\eta_2} \), where \( \eta_1 \) and \( \eta_2 \) are roots of \( f(\eta) = \frac{E}{\pi \eta} \).

For an elastic interlayer, \( \mu_1 \to 0 \) or \( m \to 0 \), we get \( \pi \to \mu_\infty \) which implies that \( \phi \to f \). Thus the right hand side of Eq. (14) approaches 0/0. Using L'Hôpital's rule, we obtain \( \omega \to 0 \). As depicted in Fig. 6, the region (II) with positive growth rates collapses gradually with decreasing \( m \) to that for a linear elastic interlayer. In region (III), it is possible that the resulting debond nucleation wavelength will depend on \( \phi \).

The growth term \( e^{\omega t} \) at the onset of softening in the CZM TS relation (i.e., point A in Fig. 2) can be written as \( e^{\omega_{\text{soften}}t} \). At a given value of \( \phi \), a higher value of the pulling rate \( \Delta \) implies a smaller amplitude of the sinusoidal oscillation that follows the onset of softening. Thus wavy debonding may not be experimentally discernible for very high pulling speeds. Since lowering the temperature implies increasing the effective relaxation time (\( \tau_e \)), lowering temperature for fixed \( \Delta \) will have similar effect as increasing \( \Delta \). This suggests the time-temperature equivalence of wavy debonding behavior and agrees with the experimental findings of Lakrout et al. [35]. Since the normalized growth rate \( \tilde{\omega} \) increases from zero to infinity as \( \phi \) is varied from \( f \) to \( (m+1)f \) for fixed \( \hbar k \), therefore, at a given \( \Delta \) and operating temperature, \( \phi \) can be increased to achieve physically discernible wavy debonding.

Recalling that \( \phi = \frac{\omega}{\Delta \tau} \), \( \alpha = \frac{\pi}{\lambda_\text{c}} \) (when \( \phi \leq \delta \leq \phi_\text{f} \)) and \( K \gg 1 \), the condition \( \phi > \phi_\text{f} = 6.22 \) gives the following requirement for the wavy debonding to ensue.

\[
K_{\text{soften}} > 4.15 K_{\text{elastic}} \tag{16}
\]

Here \( K_{\text{soften}} = \frac{E_\infty}{h} \) is a measure of the slope of the softening portion of the bilinear TS relation and \( K_{\text{elastic}} = \frac{E_0}{h} \), where \( E_\infty = 3 \mu_\infty \) is the long-term Young's modulus of the interlayer material. The quantity \( K_{\text{elastic}} \) can be thought of as the long-term stiffness of the interlayer. On the other hand, \( \phi > \phi_\text{f} \) leads to \( K_{\text{soften}} > 4.15 K_{\text{elastic}} \), where \( K_{\text{elastic}} = \frac{E_0}{h} \) and \( E_0 = 3 (\mu_\infty + \mu_1) \) is the instantaneous Young's modulus of the interlayer material. Eq. (16) suggests that for wavy debonding to ensue, the stiffness (units: N/mm\(^2\)) of the interface during the softening regime must be 4.15 times the long-term stiffness of the interlayer. Thus the TS parameters must satisfy this constraint for wavy debonding. If the softening stiffness (divided by 4.15) lies between the long-term and the instantaneous stiffness of the elastomeric layer, the dominant wavelength is expected to be independent of the CZM parameters.

Eq. (16) provides an interpretation of the debonding instability in terms of the CZM parameters. It stems from the interfacial traction decreasing with an increase in the interfacial separation, similar to what previous researchers [10,14] stated in terms of the distance-dependent van der Waals forces acting at the interface that trigger instability when a rigid adherend approaches an elastic film. Our results imply that the elastomeric layer thickness and the interfacial adhesion can be selected to either avoid or produce wavy debonding. Similarly, given the TS relation and values of parameters of the interlayer material, one can discern if the necessary condition for undulations to occur is satisfied.

One needs to analyze the nonlinear problem to study the evolution of debonding. In the following section, we use the FEM to analyze plane strain deformations of the interlayer material with the bilinear TS relation and furthermore ascertain (i) the validity of Eq. (16), and (ii) the effect of \( \phi \) on the spacing between the adjacent debonding undulations. Advantages of the FE work over the analytical work include no specific form of perturbations, using the finite size specimen, considering boundary conditions at specimen edges, not assuming a-priori the deformation field, accounting for mixed-mode debonding evolution, and the capability to simulate material and geometric nonlinearities.
3. Debonding of elastomeric layer from a rigid adherend by the FEM

3.1. Approach

The problem studied in Section 2 is now analyzed by the FEM using the commercial software ABAQUS/Standard[36]. To be consistent with the assumption made in Section 2 that the system is infinitely wide in the x direction, we use a large L/h ratio. The FE mesh and the prescribed boundary conditions are shown in Fig. 7. Five 4-node plane strain elements with hybrid formulation (CPE4H) are placed through the thickness of the interlayer. In order to characterize well the spacing between adjacent undulations, 20 elements are placed over the expected characteristic length of the undulation spacing, i.e., 3h. Results for three different FE meshes are included in Appendix C. A uniform vertical displacement, w = Δ, is applied at the rate of 1 mm s⁻¹ on the top surface of the rigid adherend while its lower smooth (frictionless) surface is bonded to the top surface of the interlayer via the CZM interaction and the bilinear TS relation[36] with mode-independent values of TS parameters. Numerical experiments are conducted by varying the parameter Ksoftening/Kelastic using several combinations of values of the interlayer modulus, μ, or μ, thickness, h, and cohesive zone parameters, K, Tc, and Gc.

3.2. Results and discussion

Wavy debonding is predicted, as shown in Fig. 8, at sufficiently large values of Ksoftening. The critical Ksoftening values plotted in Fig. 9 versus the elastic stiffness of the interlayer agree with those given by Eq. (16). The predicted debonding nucleates periodically along the x-direction implying long debonding channels along the y-direction.

Fig. 6. Effect of m: (a) elastic limit with two possible wavenumbers, (b) three regions for m=1, and (c) three regions with a wider region II for m=5.
because of the plane strain assumption. However, in probe tack tests [7] and contact experiments [2], debonding is found to nucleate periodically along both the x- and the y- directions. Motivated by the work of Huang et al. [14], we speculate that the condition for the onset of wavy debonding remains unchanged for 3D deformations of the interlayer. The simulated debonding evolution, in general, consists of the nucleation of interfacial cavities at a characteristic spacing, expansion of cavities and lateral propagation of each cavity until the adherend separates from the elastomeric layer as cavities coalesce.

Such debonding behavior in probe tack tests has been reported by Lakrout et al. [7]. Two examples of the evolution of wavy contact opening are shown in Fig. 10 for $\phi = 8.25$ and $\phi = 41.25$. The discrete Fourier transform method (available in the software MATHEMATICA [41]) is used to extract the dominant wavelength of the debond nucleation. Results are plotted in Fig. 11 for a range of $\phi$ values for an elastic interlayer ($m = 0$), and two viscoelastic interlayers with $m = 3$ and 25. The dominant wavelength is close to 3$h$ and independent of $\phi$ when $\phi$ lies in region II. This agrees with the value derived from the above analytical work and experimental findings of Mönch and Herminghaus [2]. For both the elastic and the viscoelastic layers, the wavelength to thickness ratio for $\phi$ near region III is found to be larger and dependent on $\phi$ than that when $\phi$ is near region II. It is noteworthy that at the larger $\phi$ value, shapes of the nucleated cavities change before they coalesce as was computed by Sarkar et al. [24]. Our numerical experiments suggest that the response becomes more mesh-dependent as $\phi$ is increased (see Appendix C).

Previous studies [2,9] suggest that the characteristic wavelength is independent of the adhesion seem to contradict at first sight our results for large values of $\phi$. For typical experimental [5,29] values of $m = 10^3$, $\nu = 0.04 - 0.2$ $J$/$m^2$, $\mu_1 = 0.2 - 2$ MPa, and $h = 40 - 400$ mm, we get $T_c > 2.5 \mu_1$ in the proximity of $\phi = \phi_{2c}$. Such large values of the critical traction imply probable onset of cohesive debonding mechanisms such as bulk cavitation [42] that occurs when $(-p) > 2.5\mu_1$. This suggests that the $\phi$ values for test conditions [2,9] that exhibited pure interfacial separation were well below $\phi_{2c}$ and, therefore, the dominant wavelength was indeed independent of the adhesion.

Results depicted in Fig. 9 for the interlayer material modeled as neo-Hookean do not exhibit any significant difference in the threshold softening stiffness because strains induced in the elastomer layer when softening ensues are negligible for the large value of $K$ used.

Results summarized in Table 1 indicate that wavy debonding was not predicted when either $\Delta = 100$ mm/s or a higher value of the relaxation time $\tau_{\alpha}$ was used for a given value of $\phi$. However, a lower value of $\tau_{\alpha}$ or a higher value of $\phi$ at $\Delta = 100$ mm/s resulted in wavy debonding. These results qualitatively confirm the time-temperature equivalence discussed at the end of Section 2.

4. Analysis of 3D deformations for peeling of a flexible plate off a soft elastomeric layer by the FEM

4.1. Approach

Results presented in the preceding two sections imply that values of parameters in the TS relation for the interface between an elastomeric layer and the rigid substrate, the applied pulling rate and the characteristic relaxation time determine whether or not a wavy debond will occur. In order to delineate fingering instability [4,5,9] for a deformable upper adherend, we study progressive crack propagation in the configuration of Fig. 12 that resembles the test set up often used to characterize interfacial adhesion and/or study mechanics of interfacial separation [5,29,43]. Major differences between this problem and those studied above include bending and stretching deformations of the upper adherend.

We model the plate/adherend and the interlayer of dimensions, respectively, $20 \times 30 \times 1$ mm$^3$ and $20 \times 25 \times h$ mm$^3$. Deformations of the interlayer may be large but those of the adherend plate are assumed to be infinitesimal. Furthermore, inertial effects are neglected. We use values of the material parameters given by Murray [43] who experimentally observed fingering like debonding during the peeling of a glassy polymer plate from a hydrogel interlayer in a wedge test. The plate material is modeled as homogeneous and isotropic Hookean.
Fig. 10. Evolution of the contact opening for $m = 3$ and (a) $\phi = 8.25$ ($\phi_1 < \phi < \phi_2$) and (b) $\phi = 41.25$ ($\phi > \phi_2$). Variables in the plot are: $\delta = \delta_f/\delta_0$ and $x = 2x/L$. The deformation shown is multiplied by 5 for ease in visualization. Plots correspond to the central portion of the interface of non-dimensional length 1. Deformed configurations are for the interface portion of non-dimensional length 0.66.
with Young’s modulus, \( E_{\text{plate}} = 2.1 \) GPa and Poisson’s ratio = 0.4. However, we also compute results for different values of \( E_{\text{plate}} \). The material for hydrogel interlayer is modeled as isotropic, homogeneous and incompressible with the constitutive relation \((17)\) (e.g., see Simo [44]) implemented in ABAQUS [36].

\[
\sigma(t) = -p1 + a_1B + a_2B^2 + d \int_0^t g_R(s) F_t^{-1}(t-s)\sigma_0(t-s)F_t^{-1}(t-s) ds
\]

(17)

Here \( a_1 \) and \( a_2 \) are material constants, \( B \) the left Cauchy–Green tensor, \( g_R \) the normalized relaxation modulus expressed as Prony series \( g_R(t) = \mu_R(t) / \mu_R(0) \), \( \text{dev}(A) = A - \frac{1}{3} \text{tr}(A)I \), \( \text{tr}(A) = \text{sum of the diagonal elements of the matrix } A \) when its components are written with respect to an orthonormal basis, \( I = \text{the identity matrix} \), \( \sigma_0 \) the instantaneous deviatoric Cauchy stress, and \( F_t \) the deformation gradient at time \((t-s)\) with respect to the configuration at time \( (t) \), defined as \( F_t(t-s) = \frac{\partial \mathbf{x}}{\partial \mathbf{x}_0} \) with \( \mathbf{x} \) giving the current position of a material point. Based on experimental results [45], the instantaneous elastic response is assumed to be neo-Hookean.

Table 1: Qualitative evidence of the time–temperature equivalence in predictions from the model.

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( \Delta ) (mm/s)</th>
<th>( \sigma_0 ) (( \mu )s)</th>
<th>Debonding (Deformations exaggerated by a factor of 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>100</td>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Effect of the TS parameters (for plate rigidity = 2.1 Nm, interlayer thickness = 250 \( \mu \)m, and pulling rate = 1 mm/s), which evince that wavy undulations vanish when the interfacial softening is increased relative to the interlayer stiffness (lower effective stiffness at a higher temperature due to viscoelasticity).

<table>
<thead>
<tr>
<th>( \tilde{T} ) (C)</th>
<th>( T_0 ) (MPa)</th>
<th>( G_c ) (J/m(^2))</th>
<th>Type of debonding front</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>0.04</td>
<td>0.2</td>
<td>Fingerlike undulations</td>
</tr>
<tr>
<td>55</td>
<td>0.004</td>
<td>0.2</td>
<td>No fingerlike undulation</td>
</tr>
<tr>
<td>-30</td>
<td>0.04</td>
<td>0.2</td>
<td>No fingerlike undulation</td>
</tr>
</tbody>
</table>

Fig. 11. Effect of \( \phi \) on the dominant wavelength (normalized by thickness) of the debonding undulation.

Fig. 12. The FE mesh and boundary conditions with zero tractions along unconstrained displacement components not exhibited.
i.e., \( a_2 = 0 \), and \( a_1 = \text{twice the constant shear modulus of the interlayer material. The Prony series parameters determined experimentally [45] at the reference temperature of 45 °C are listed in Table B1. The WLF constants at the reference temperature \( T_{REF} = 45 \) °C are \( C_1 = 4.57 \) °C and \( C_2 = 142.2 \) °C. We assume a mixed-mode bilinear TS relation [36] with mode-independent values of the initial slope \( K = 10^6 \) MPa \( \cdot \) mm, the peak traction \( (T_f = 0.04 \) MPa), and the fracture energy \( (G_c = 0.2 J/m^2) \). The value of the damage stabilization parameter for alleviating numerical instabilities was gradually reduced to \( 10^{-10} \) until a further decrease in this value did not affect computed results.

The boundary conditions and the FE mesh are shown in Fig. 12. The three displacement components of points on the lower face of the interlayer are set equal to zero to simulate its infinitely strong bonding to the stationary rigid adherend. The same vertical displacement \( \Delta \) is applied at 1 mm \( s^{-1} \) to all nodes on the plate right edge. Tactions on the other bounding surfaces and tangential tractions on the right edge are null.

The eight-node brick elements (C3D8R) with reduced integration and the default hourglass control option have been used for the cover plate. Hybrid elements (C3D8RH)7 have been employed for the incompressible interlayer. The interlayer region is discretized into 5 (thickness) \( \times 150 \) (width) \( \times 100 \) (length) uniform FE mesh. This FE mesh is reasonably fine as the characteristic spacing between adjacent undulations is expected [5] to be \( \lambda = 3h - 4h = 1 \) mm for the smallest interlayer thickness used in the study. In order to demonstrate the effect of the CZM parameters on the debonding characteristics, we have not obtained a fully converged solution by successively refining the FE mesh since qualitative features of progressive debonding remained independent of the FE mesh.

4.2. Results and discussion

In general, the debond front is either fingerlike or straight as illustrated in Fig. 13. An example of the evolution of the fingerlike debonding process is shown in Fig. 14 in which we have plotted contours of the contact opening \( \delta \) at three different times for a plate of bending rigidity, \( D = \frac{E_{plate} h_{plate}^3}{12(1-\nu^2)} = 21 \) Nm (\( \nu = \text{Poisson's ratio}, h_{plate} = \text{the plate thickness} \) and an interlayer of thickness 250 \( \mu \)m.

Computed values of \( \delta > \delta_f \) are represented in red color at right ends of illustration. As experimentally observed [5,9], the development of fingers is preceded by the nucleation of debonds spaced at approximately the same distance as that between the fingers that will ensue. Decaying plate displacement, undulatory debonding, and lateral propagation of debonds give rise to what resembles fingers at the debond front.

Progressive debonding was also simulated for a plate with \( D = 2.1 \) Nm and interlayers thicknesses 250 \( \mu \)m, 500 \( \mu \)m, 600 \( \mu \)m and 750 \( \mu \)m. The spacing between the adjacent fingers, computed using the discrete Fourier transform method (using MATHEMATIC [41]), versus the interlayer thickness is plotted in Fig. 15a. We note that the computed spacing increases with an increase in the interlayer thickness and reasonably agrees with experimental findings of Ghatak and Chaudhury [9] who reported the approximate relation: \( \lambda \approx 4h \). We note that for the highest thickness (750 \( \mu \)m) simulated, fingerlike debonding was not predicted. However, debonding exhibited a fingerlike front for a plate of higher flexural rigidity (7.5 Nm), consistent with the requirement of a threshold lateral confinement \( \left( D/\mu h^3 \right)^{1/3} \), expressed as the ratio of the two length scales, \( (D/\mu)^{1/3} \) and \( h \) [9,12]. Simulations conducted for different values of \( D \) and a constant interlayer thickness (250 \( \mu \)m) revealed that the length of the fingers increased monotonically with the quantity \( (D/\mu)^{1/3} \) which represents the characteristic stress decay distance for a flexible plate bonded to an elastomeric foundation [46]. The linear fit to the computed values depicted in Fig. 15b is close to that obtained by Ghatak and Chaudhury [9] for a different material system. We believe that this quantitative agreement in the finger amplitudes is coincidental because the softening zone length (in the \( y \)-direction) should also depend on the interfacial adhesion. We found that the dominant frequency of undulation is relatively insensitive to the plate rigidity, consistent with findings of [9].

In order to qualitatively demonstrate the concept that the CZM parameters for confined interlayers determine whether or not contact separation resulting from tension is wavy, additional simulations were conducted by varying the TS parameters and the operating temperature by keeping the thickness of the interlayer, the plate rigidity and the pulling rate constant. The key parameters used in these simulations and the results are summarized in Table 2. For \( T_c = 0.04 \) MPa, setting the temperature equal to \(-30 ^\circ C\) results in larger relaxation times following the WLF equation, and a fingerlike debond front was not predicted. Similarly, a fingerlike

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7 For the 3D simulations, the overall energy balance applied to the ABAQUUS output gave a discrepancy of about 20%. Additional numerical experiments revealed that this was due to our using the default value of the hourglass control. The use of selective reduced integration or other hourglass control options satisfied the energy balance within 1% error but triggered unphysical oscillations at the contact surface. The physically meaningful results computed with the default hourglass control and their agreement with the literature results obtained by other methods provide credence to results reported here.

8 The shear modulus of the interlayer at time \( t \) and temperature \( T \) was estimated as \( \mu(t, T) = \mu_0 \left( \frac{T}{T_{REF}} \right)^{C_1/C_2} = \mu_0 + \sum_{i=1}^{n} \mu_i e^{-\frac{T}{T_i}} \)
front was not predicted when $T_c = 0.004 \text{ MPa}$ was used for the operating temperature of 55°C. Our computed results suggest that the debonding behavior during peeling a flexible plate off an elastomeric layer constrained to a rigid base is dictated by both the lateral confinement $D = \frac{\mu h^3}{C_1 C_0}$ and the adhesion parameter, $T_c^2 h G_c E_1^{4/3}$. A detailed analysis probing their collective role in the debonding evolution will be undertaken in a future work.

5. Conclusions

We have studied debonding of a confined elastomer layer from an adherend using the cohesive zone model (CZM) and the bilinear traction-separation (TS) relation for the interaction between the elastomer layer and the adjoining adherend. The stability analysis of the homogeneous solution (null displacements and constant hydrostatic pressure) of plane strain deformations of the elastomer and the analysis of deformations by the finite element method (FEM) have enabled us to conclude that a necessary condition for a wavy/undulatory debonding to ensue is $\frac{T_c^2 h}{G_c E_1^{4/3}} > 4.15$, where $T_c$ is the peak traction and $G_c$ the fracture energy in the TS relation, $h$ the thickness and $E_1$ the long-term Young’s modulus of the elastomeric layer modeled as linear viscoelastic. This result can help design a material system for avoiding wavy debonding by choosing thickness of the soft adhesive and/or altering the interfacial softening by suitable surface treatment. It also serves to tailor the TS relation parameters for simulating spatially undulatory debonding evolution. The linear stability analysis also predicts that if undulations occur, their dominant wavelength is close to $3h$, when $T_c^2 / 4.15 G_c$ lies between $E_\infty / h$ and $E_0 / h$ where $E_0$ is the instantaneous modulus of the elastomeric layer. Analysis of the problem using the finite element method (FEM) provides details of the interfacial debonding evolution and sheds light on the effects of pulling rate and temperature.

We have also analyzed using the FEM three-dimensional deformations of a thin elastomeric interlayer (perfectly bonded to a rigid base) when a flexible plate is peeled from it by applying vertical displacements to points on one edge. This analysis predicts the progressive debonding with a fingerlike front is controlled by both the lateral confinement $\left(\frac{D}{\mu h^3}\right)^{1/3}$ and the adhesion...
parameter, $\phi$, where $D$ equals the plate bending rigidity and $\mu$ the interlayer shear modulus at the operating temperature.

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**Appendix A**

Using the non-dimensionalization $X = x/h, Z = z/h, U = u/h, W = W/h, P = P/T_c$ and $T = t/t_{int}$ where $t_{int} = \Delta / \Delta$ is the time of interest, equations of motion for the interlayer become

$$\frac{\partial^2 U}{\partial T^2} + \left( \frac{t_{int}}{twave} \right)^2 \left( -\frac{\partial P}{\partial X} + \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial X^2} \right) \right) = 0$$

$$\frac{\partial^2 W}{\partial T^2} + \left( \frac{t_{int}}{twave} \right)^2 \left( -\frac{\partial P}{\partial X} + \left( \frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial X^2} \right) \right) = 0$$

where $twave = h/\sqrt{\mu}$ equals the time for an elastic wave to travel through the thickness, $h$, of the interlayer material of mass density $\rho$. For typical representative values ($m \approx 10^4$, $\omega \approx 0 - \infty$, $\phi \approx 4 - 4000$, $\Delta \approx 1$ mm/s, $\mu_c \approx 0.4 - 0.2$ J/m$^2$, $h \approx 40 - 400$ mm, and $\rho \approx 1$ kg/m$^3$), $(t_{int}/twave)^2 = \frac{4}{\mu} \left( \frac{h^2}{2} \right) \left( 1 + \frac{m}{\mu} \right) > 1$. Thus inertia terms are negligible. However, the inertia term will potentially play a significant role for faster pulling speeds for which $(t_{int}/twave)^2$ is not much greater than 1.

**Appendix B**

See appendix Table B1.

**Appendix C**

As suggested by a reviewer we include here the effect of the FE mesh on results of the plane strain problem studied in Section 3. Sensitivities of the load-displacement histories and the spatial variations of the contact opening for three FE meshes in the interlayer are examined. The FE meshes 1, 2, and 3 had, respectively, 5, 5, and 8 elements through the thickness with element widths of 0.05, 0.04 and 0.03 mm. Results plotted in Fig. C1 and Fig. C2 for $\phi = 8.25$ and $\phi = 41.25$, respectively, are sensitive to the FE mesh at the larger value of $\phi$. A coarse mesh causes spurious oscillations as can be seen from Fig. C2b. However, discrete Fourier transforms of the computed contact openings for mesh 2 and mesh 3 (Fig. C2a) yield the same dominant frequency ($\approx 41h$).

<table>
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<td>10</td>
<td>$10^3$</td>
<td>$10^4$</td>
<td>$10^5$</td>
<td>$10^6$</td>
</tr>
</tbody>
</table>

![Fig. C1](image1.png) **Fig. C1.** For $\phi = 8.25$, sensitivity to the FE mesh of (a) the dimensionless reaction force ($\bar{P} = R/T_c$) vs. the dimensionless displacement, $\bar{X}$, applied at a rate of 1 mm/s when $m = 3$, and (b) the dimensionless interfacial contact opening when $\Delta = 0.54$.

![Fig. C2](image2.png) **Fig. C2.** For $\phi = 41.25$, sensitivity to the FE mesh of (a) the dimensionless reaction force vs. dimensionless displacement applied at a rate of 1 mm/s when $m = 3$, and (b) the dimensionless interfacial contact opening when $\Delta = 1.67$. 

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References


