On The Inharmonicity Of A String

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The phenomenon called the inharmonicity is generated for a struck string, typically a piano string. It is known that its spectral components consist of two inharmonic peak series: “the first partials” due to the elastic stiffness of a string and “the second partials”. Many papers (1)−(3) have described that the tones of a piano involve other strange partials, “the second partials” which Conklin (2) called “phantom partials”.

Experimental tests for examining a mechanism of the second partials were performed for struck and plucked strings. Plain electric guitar strings from 0.010 to 0.018 inch in diameter $D$ were used for inharmonicity tests and made of high carbon steel. The string length $l$ was 510.5 mm. The string was struck with a hammer head having the restoring force of a spring with initial displacement $d_0$. The striking point was at a distance of 154 mm from the bridge. On the other hand, the string was plucked at the same point as the striking one by cutting a thin thread which lifts the string. The observation point was located at 25 mm from the nut. Transverse string displacements were measured by means of the eddy-current type gap sensor. The string tension was tuned to the fundamental natural frequency, $160 \pm 5$ Hz, by the peg.

In this paper, we describe experimental results only for a plucked string, while ones for a struck spring have already been shown in our paper (4). Figure 1 shows a time history of string displacement and its frequency spectrum by FFT through Hanning window function. Here, peaks lower than $-80$ dB in level are omitted as noise, where $1V=0$ dB $=0.2$ mm. Black and white circles show the first and the second partials, respectively. We can see that peaks lower than the first partial in level appear below the frequencies of the first partials. Their dimensionless differences $d_i = (f_n - n f_0)/f_0$ are shown in Fig.2 versus partial number $n$, where $f_0$ : the fundamental natural frequency and $f_n$ : $n$-th frequency of the first partials. Amplitudes in the range $0$ dB to $-80$ dB are classified each $10$ dB by shaded circles. That is, the darker colored circles represent the larger amplitude. A solid line shows $d_i$ calculated by $B n^3/2$, where the value of the inharmonic coefficient $B$ was estimated from measured natural frequencies. A dotted line shows $d_i$ by substituting $1/4$ of the value of $B$. It is found that many low peaks in level appear besides the first and second partials. In particular, we shall discuss peaks which appear at frequencies below and above the 11th frequency of the first partial, $1811$ Hz. The frequencies of the first partials $1$−$8$ are respectively $161$ Hz, $322$ Hz, $483$ Hz, $645$ Hz, $809$ Hz, $911$ Hz, $1137$ Hz and $1301$ Hz. These peaks have partials that
A struck spring

Figures 3(a) and (b) show frequency spectra of transverse and longitudinal displacements ($w$ and $u$) calculated by Eqs. (1) and (2) for a struck string.

Fig. 3 Frequency spectra of transverse and longitudinal displacements ($w$ and $u$) calculated by Eqs. (1) and (2) for a struck string.

The second partials are close to peaks in the longitudinal displacement. Simulated dimensionless differences $d_i$ between peak frequency and the harmonic partial frequency are plotted in Fig. 4. In this figure, peak frequencies below and above the 10th first partial are written. These peaks have partials that have frequencies equal to the sum or the difference of the frequencies of a pair of the first partials as shown in experimental results. However, in lower mode, a few peaks different from these peaks appear.

Experiments performed on the second partials of inharmonicity reveal that many peaks appear at the sum or the difference of the frequencies of a pair of the first partials in frequency spectra of transverse string displacement. The numerical simulations show that a large amplitude of string produces the second partials, and that in addition to the second partials, a few frequency components appear due to the newly developed nonlinear wave equation unlike in experimental results. Consequently, further complete explanation must be provided regarding a mechanism of the generation of the second partials. This is our future work.

Fig. 4 Calculated dimensionless difference $d_i$ of frequencies for $D : 0.016$ inch, $f_0 : 160$ Hz, impact force $F_s : 1.8$ N, contact time $\tau : 18$ msec.

References