% 3-D vortex-lattice method for a rectangular wing
clc
clear

% span and chord
span = 10
chord = 5

% numbers of rows and columns
nr = 6
nc = 12

% angle of attack
alpha = 10

% number of steps to be run
nsteps = 1

sa = sind(alpha);
ca = cosd(alpha);

nrp = nr + 1;
ncp = nc + 1;

% grid sizes
dy = span/nc;
dx = chord/nr;

% grid set-up
for i = 1:nrp
    for j = 1:ncp
        x(i,j) = (i-1)*dx;
y(i,j) = (j-1)*dy;
z(i,j) = 0;
    end
end

% coordinates for plotting
for j = 1:ncp
    xplt(j) = x(1,j);
yplt(j) = y(1,j);
end

% check the grid
% note this figure continues beyond control points
figure(1)
plot( yplt, xplt,'r' )
hold on

for i = 2:nrp
    for j = 1:ncp
        xplt(j) = x(i,j);
yplt(j) = y(i,j);
    end
    plot( yplt, xplt,'r' )
end

for j = 1:ncp
    for i = 1:nrp
        xplt(i) = x(i,j);
yplt(i) = y(i,j);
    end
    plot( yplt, xplt,'r' )
end
axis equal

% number the elements and define the control points
% and vectors normal to the elements
n = 0;
for i = 1:nr
    for j = 1:nc
        n = n + 1;
    end
end
% control points

\[
xcp(n) = 0.25*(x(i,j) + x(i,j+1) + x(i+1,j+1) + x(i+1,j));
\]
\[
ycp(n) = 0.25*(y(i,j) + y(i,j+1) + y(i+1,j+1) + y(i+1,j));
\]
\[
zc\(n\) = 0.25*(z(i,j) + z(i,j+1) + z(i+1,j+1) + z(i+1,j));
\]

% the normal vectors

\[
d1x = x(i+1,j+1) - x(i,j);
\]
\[
d1y = y(i+1,j+1) - y(i,j);
\]
\[
d1z = z(i+1,j+1) - z(i,j);
\]
\[
d2x = x(i,j+1) - x(i+1,j);
\]
\[
d2y = y(i,j+1) - y(i+1,j);
\]
\[
d2z = z(i,j+1) - z(i+1,j);
\]
\[
nx(n) = d1y*d2z - d1z*d2y;
\]
\[
ny(n) = d1z*d2x - d1x*d2z;
\]
\[
nz(n) = d1x*d2y - d1y*d2x;
\]
end
end

% total number of elements: nel = nr*nc; check?

nel = n;

% plot control points on grid
plot( ycp, xcp,'+b')
axis([-1,11, -1, 6])
hold off

% influence matrix

for nrec = 1:nel
xp = xcp(nrec);
yp = ycp(nrec);
zp = zcp(nrec);

for i = 1:nr
for j = 1:nc
x1 = x(i,j);
y1 = y(i,j);
z1 = z(i,j);

x2 = x(i,j+1);
y2 = y(i,j+1);
z2 = z(i,j+1);

[ u, v, w ] = BSL( x1,y1,z1, x2,y2,z2, xp,yp,zp );
s(i,j) = nx(nrec)*u + ny(nrec)*v + nz(nrec)*w;
end
end

for i = 1:nr
for j = 1:ncp
x1 = x(i,j);
y1 = y(i,j);
z1 = z(i,j);

x2 = x(i+1,j);
y2 = y(i+1,j);
z2 = z(i+1,j);

[ u, v, w ] = BSL_A( x1,y1,z1, x2,y2,z2, xp,yp,zp );
c(i,j) = nx(nrec)*u + ny(nrec)*v + nz(nrec)*w;
end
end
nsen = 0;
for i = 1:nr
    for j = 1:nc
        nsen = nsen +1;
        A(nrec, nsen) = s(i,j) + c(i,j+1) - s(i+1,j) - c(i,j);
    end
end
for n = 1:nel
    R(n) = -ca*nx(n) - sa*nz(n);
end
G = inv(A)*R';

% circulations around the vortex segments
% spanwise segments
gs(1:nc) = G(1:nc);
for n = (nc+1):(nr*nc)
    gs(n) = G(n) - G(n-nc);
end
for n = (nel+1):(nr*(nc+1))
    gs(n) = -G(n-nel);
end

% chordwise segments
n = 0;
nch = 0;
for i = 1:nr
    n = n + 1;
nch = nch +1;
gc(nch) = G(n);
    for j = 2:nc
        n = n+1;
nch = nch+1;
gc(nch) = G(n)-G(n-1);
    end
end
nch = nch+1;
gc(nch) = -G(n);
% shed the vorticity along the tips and trailing edge into the wake

for time = 1:nsteps
  % starboard wing tip
  for i = 1:nr
    n = (i-1)*nc + 1;
    xw(i,time) = x(i,1);
    yw(i,time) = y(i,1);
    zw(i,time) = z(i,1);
    Gw(i,time) = G(n);
  end

  % trailing edge
  n = (nr-1)*nc + 1;
  for j = 2:nc-1
    n = n + 1;
    xw(nr-1+j, time) = x(nrp,j);
    yw(nr-1+j, time) = y(nrp,j);
    zw(nr-1+j, time) = z(nrp,j);
    Gw(nr-1+j, time) = G(n);
  end
  xw(nr+nc-1, time) = x(nrp,nc);
  yw(nr+nc-1, time) = y(nrp,nc);
  zw(nr+nc-1, time) = z(nrp,nc);

  % port wing tip
  for k = 1:nr
    n = nr*nc - (k-1)*nc;
    xw(nr-1+nc+k, time) = x(nrp-k, ncp);
    yw(nr-1+nc+k, time) = y(nrp-k, ncp);
    zw(nr-1+nc+k, time) = z(nrp-k, ncp);
    Gw(nr-2+nc+k, time) = G(n);
  end
end

Comparison: Center Span 3-D, AR = 6 and 2-D
Immediately after an impulsive start, $\alpha = 10^\circ$
spanwise circulations

<table>
<thead>
<tr>
<th></th>
<th>3-D</th>
<th>2-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35058</td>
<td>0.095984</td>
<td>0.035546</td>
</tr>
<tr>
<td>0.035546</td>
<td>1.6653e-016</td>
<td>-0.035546</td>
</tr>
<tr>
<td>-0.095984</td>
<td>-0.35058</td>
<td></td>
</tr>
</tbody>
</table>

movie: “one”
COMBINING THE MODELS

Structural Grid

Aerodynamic Grid

Aerodynamic Panel \( j \)

Structural Node \( k \)

Aerodynamic Node \( i \)

(C.P.) \( j \)

\( \vec{F} \) \( A \)

\( u_1^k, \theta_1^k \)

\( u_2^k, \theta_2^k \)

\( u_3^k, \theta_3^k \)
The finite element method is used to discretize the equations. The beam is represented by so-called “C-bar” elements, which have six degrees of freedom at each node:

movies: “two”, “three”, “four”
Time-Domain Nonlinear Aeroelastic Analysis with Discrete-Gust Excitation:

A HALE-Wing Case Study

Z. Wang  
P. C. Chen  
D. D. Liu

D. T. Mook  
M. J. Patil

ZONA TECHNOLOGY

Virginia Tech

VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
movie: “five”
June 26, 2003, 10:45a local time

Movie: “… fishin”