Solution of a Two Degree of Freedom, damped, forced system having Cubic Nonlinearities by using Jacobi Elliptic Functions.

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Abstract

In this work, we present analytical results of a system of two coupled, forced, damped, ordinary differential equations having cubic nonlinearities with sinusoidal driving force of the form:

\[
\begin{bmatrix}
\dot{u}_1 \\
\dot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
v_1 & v_2 & \omega_{n1}^2 & 0 \\
v_2 & v_3 & 0 & \omega_{n2}^2
\end{bmatrix}
\begin{bmatrix}
\dot{u}_1 \\
\dot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
\phi_1 u_1^3 + \phi_2 u_1^2 u_2 + \phi_3 u_1 u_2^2 + \phi_4 u_2^3 \\
\phi_5 u_1^3 + \phi_6 u_1^2 u_2 + \phi_7 u_1 u_2^2 + \phi_8 u_2^3
\end{bmatrix}
= \begin{bmatrix}
F_1 \\
F_2
\end{bmatrix}
\cos \omega_f t
\]

where the dot denotes the time derivative, \( u_1 \) and \( u_2 \) are known as the system normal coordinates, \( \varepsilon \) is a non-linear parameter, \( \omega_f \) is the driving frequency, \( v_1, v_2, v_3, \omega_{n1}, \omega_{n2}, F_1, F_2, \) and \( \phi_1 \) through \( \phi_8 \) are corresponding system parameters. This type of equations is associated with many physical systems such as the vibration of strings, beams, absorbers, plates, and so on. It is shown that the analytical general evolution approximate solution is given as a combination of Jacobi elliptic functions and trigonometric ones [1] – [3]. The degree of accuracy between our derived analytical solution and the numerical integration solution is illustrated for the system shown in Figure 1.

![Nonlinear mechanical system acted upon external forces.](image)

Results

Comparison of our derived solution with the numerical integration solution of Equation (1) obtained by using the Fourth Order Runge-Kutta numerical algorithm provided my Mathematica 6 is shown in
Figures 2 and 3. In these Figures, the solid lines represent the numerical integration solutions and the dashed lines represent the proposed solution. For the system parameters shown in the caption of Figures 2 and 3, we may see that the derived analytical solution based on Jacobi elliptic function compares well with the numerical integration solution for either small or moderate values of the nonlinear parameter $\varepsilon$.

Figure 2. Amplitude-time response curves. The parameter values are:

$k_1 = 3, \quad k_2 = 3, \quad Q = 1, \quad c = 0.1, \quad \varepsilon = 1, \quad \omega_f = \omega_{n2} = 3, \quad x_1(0) = 1, \quad \text{and} \quad x_2(0) = 0.5.$

Figure 3. Amplitude-time response curves. The parameter values are:

$k_1 = 3, \quad k_2 = 1, \quad c = 0.01, \quad Q = 1, \quad \varepsilon = 0.5, \quad \omega_f = (\omega_{n1} + \omega_{n2})/2 = 1.68, \quad x_1(0) = 1, \quad \text{and} \quad x_2(0) = 1.$

References

