Study of Contact-Impact Force Models in Multibody Mechanical Systems

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Abstract

This work deals with the contact-impact force models for spherical and cylindrical shaped surface collisions in multibody mechanical systems. The impact phenomenon is characterized by abrupt changes in the values of system variables, most commonly discontinuities in the system velocities. Other effects directly related to the impact phenomena are those of vibration propagation through the system, local elastic/plastic deformations at the contact zone and frictional energy dissipation. Impact is a prominent phenomenon in many mechanical systems such as mechanisms with intermittent motion and mechanisms with clearance joints [1,2]. Therefore, in order to correctly simulate and design these kinds of mechanical systems adequately, some appropriate contact-impact force model must be adopted.

In a broad sense, there are two different methods to solve the impact problem in multibody systems, namely, the continuous and discontinuous approaches [3]. Within the continuous approach the methods commonly used are the continuous force model, which is in fact a penalty method, and the unilateral constraint methodology, based on complementary approaches [4]. The continuous contact force model represents the forces arising from collisions, and assumes that the forces and deformations vary in a continuous manner. In the discontinuous method it is assumed that the impact occurs instantaneously and the integration of the equations of motion is halted at the time of impact. In the discontinuous method, the dynamic analysis of the system is divided into two intervals, before and after impact. The restitution coefficient is employed to quantify the dissipation energy during the impact. This method is commonly referred to as piecewise analysis, and has been used for solving the intermittent motion problem [1].

The best-known force model between two spheres of isotropic materials is the non-linear Hertz law [5],

\[ F_N = K\delta^n \]  

(1)

where \( K \) is the generalized stiffness constant and \( \delta \) is the relative normal indentation between the spheres. The exponent \( n \) is set to 1.5 for metallic surfaces [5]. Lankarani and Nikravesh [3] developed a contact force model with hysteresis damping for impact in multibody systems. This model uses the general trend of the Hertz contact law, in which a hysteresis damping function was incorporated in the model that represents the energy dissipated during the impact. This model can be expressed as,

\[ F_N = K\delta^n \left[ 1 + \frac{3(1-e^2)}{4} \frac{\delta}{\delta^{(-)}} \right] \]  

(2)

where \( K \) is the generalized stiffness constant, \( e \) is the restitution coefficient, \( \delta \) is the relative penetration velocity and \( \delta^{(-)} \) is the initial impact velocity.

The contact force models given by Equations (1) and (2) are only valid for colliding bodies with circular contact areas. For a cylindrical contact area between two parallel cylinders, a literature search has revealed few and approximate force-displacement relationships. Some authors [6] suggest the using of the more general force-displacement relation given by Equation (2) but with a lower exponent, \( n \), between 1 and 1.5.

Based on the Hertz theory, Dubowsky and Freudenstein [7] presented an expression for indentation as a function of the force on the internal pin inside a cylinder as,

\[ \delta = F_N \left( \frac{\sigma_i + \sigma_j}{L} \right) \ln \left( \frac{L_n(R_j - R_i)}{F_N R_j (\sigma_i + \sigma_j)} \right) + 1 \]  

(3)

where \( R_{ij} \) are the radii of contact bodies, \( L \) is the length of the cylinder, \( \sigma_{ij} \) are parameters that depend on the material properties, and the exponent \( m \) has a value of 3. Since Equation (3) is nonlinear implicit function for \( F_N \), with a known penetration depth, \( \delta \), \( F_N \) can be evaluated. This being a nonlinear problem, an iterative scheme is
necessary to solve for the normal contact force $F_N$. Goldsmith [8] presented an expression similar to Equation (3) but with the exponent value of $m$ equal to 1. This value, however, leads to problems with the units in the expression. The ESDU-78035 Tribology Series [9] presented some expressions for contact mechanics analysis suitable for engineers’ applications. For a circular contact area the ESDU-78035 model is the same as the pure Hertz law, given by Equation (1). For rectangular contact, e.g., a pin inside a cylinder, the expression is,

$$
\delta = F_N \left( \frac{\sigma_1 + \sigma_3}{L} \right) \left[ \ln \left( \frac{4L(R_1 - R_2)}{F_N(\sigma_1 + \sigma_3)} \right) + 1 \right] 
$$

(4)

A comparison between the circular and cylindrical contact force models are presented in Figure 1.

Some important conclusions can be drawn from the study presented in this work. The Hertz relation besides its nonlinearity does not account for the energy dissipation during the impact process. The contact models for cylindrical contact areas do not present any advantage compared to the contact spherical models. Moreover, the cylindrical models are nonlinear and implicit functions, requiring a numerical iterative procedure. Furthermore, these models are purely elastic in nature and cannot explain the energy dissipation during the impact process. Therefore, the Hertz relation along with the modification to explain the energy dissipation in the form of internal damping can be adopted for modeling contact forces in a multibody system. From the comparison between the cylindrical contact force models and circular contact force models (Figure 1), one can conclude that the spherical and cylindrical force-displacement relations are reasonably close. Further, the straightforward force-penetration relation proposed by Lankarani and Nikravesh is largely used for mechanical contacts owing to its simplicity and easiness of implementation in a computational program, and also because this is the only model that accounts for energy dissipation during the impact process.

Acknowledgments

The work presented here was supported by Fundação para a Ciência e a Tecnologia and partially financed by Fundo Comunitário Europeu FEDER within the project Sapiens Nr. 38281 entitled ‘Dynamic of Mechanical Systems with Joint Clearances and Imperfections’.

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