Analysis of elastic stress wave propagation through a complex composite structure

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Abstract: A one-dimensional impact problem is employed for a layered system that comprises heterogeneous materials with different geometrical configurations. The system mimics transmitter for Multiple Integrated Laser Engagement Systems (MILES). The analytical model tracks the stress wave propagation through this layered structure as a result of the mechanical shock from blank rounds as they fired. The results clearly support the hypothesis that layered systems will suffer stress amplification with inherent acoustic impedance mismatch between layers. It is evident that the time dependent stress response in the rise portion is significantly affected by impedance mismatch and the number of layers.

Keywords: stress wave; impedance; layered composites; acoustic impedance.


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1 Introduction

With functionally graded materials increasingly used in systems and structures designed to function in the severe shock environments, the assessment and prediction of the
response of those systems to complex loading conditions is crucial. Shock loading to a solid body can be induced through high velocity planer impact or high energy explosive detonation (Asay and Shahinpoor, 1993; Davason and Graham, 1979). There are several standard test configurations for generating dynamic loadings, such as drop weight test, Hopkins bar impact, plate impact and projectile impact tests.

Roughly, the impulsive shock loading may be categorised into three regimes: Strong shock or high pressure, weak shock or intermediate pressure and elastic or low pressure; the corresponding behaviour of the solids are respectively hydrodynamic, finite-strain plastic and linear elastic (Graham, 1993). In the strong shock loading regime, stress is several times larger than the yield stress of the material and thus the material strength effects may be neglected and the solid may be treated as an inviscid compressible fluid, which is referred to as hydrodynamics (Zukas, 2004).

In elastic regime, upon loading, the material deforms in a completely reversible manner. The response of the material is described by linear elastic constitutive laws; Hooke’s law. Due to the linearity of the governing equations and the constitutive relations, most dynamic problems in the elastic regime can be analytically solved. Therefore, the theories on elastic wave propagation in solids are mostly fully developed, (Achenbach, 1973; Bedford and Drumheller, 1994).

Most of the work on stress wave propagation was carried out for homogeneous materials. Several studies expanded the analysis to heterogeneous materials. In particular several experiments have been performed that concern the finite-amplitude wave propagation in composite materials. An ideal system to investigate the wave propagation in laminated composites is periodically layered media.

Wave dispersion in periodic elastic layered systems has been extensively studied. Rytov (1968) obtained a dispersion relation for one-dimensional longitudinal waves propagating in a periodic laminate. Later Sun et al. (1968) obtained the dispersion relations for harmonic wave propagating parallel to and normal to the direction of the layering, using the effective stiffness theory for a unit step loading at the boundary.

Barker et al. (1974) proposed a general nonlinear Maxwell (viscous) model to simulate stress relaxation from an instantaneous state of the mixture to the equilibrium level. In addition, Barker obtained oscillatory stress solutions by explicitly modelling each layer using a stress wave propagation based one-dimensional code. Barker successfully validated the viscous model for the composite equation of state through matching the averaged stress response in the oscillations from the code. Barker found that below certain critical input amplitude, the stress wave amplitude decayed exponentially with distance away from the source and formed a structured shock wave above the critical amplitude.

Oved et al. (1978) also conducted limited shock wave experiments on layered stacks of Cu/PMMA which revealed resonance phenomena due to layering. The oscillations occur about a mean value (called mean stress). When the amplitude of oscillations is substantial, Oved pointed out that oscillations do not vanish with distance of propagation in the shock region. Consequently, oscillations should not be ignored but should be explicitly modelled.
Several models for estimating the history and profile of shock wave propagating in composites have been proposed. Barker (1971) proposed viscous type equation of state to describe the stress wave propagation in layered composites, in which the dispersive effects due to interactions of wave with interfaces and reflected waves are accounted for by direct analogy with the viscosity effect existing in the viscoelastic materials.

Wave propagation in the media with periodically arranged spherical particles have been rigorously formulated and studied using Floquet theory (Achenbach and Kitahara, 1987). In this research, since we restrict ourselves to the planar layered heterogeneous system (motivated by the applications of engineering laminates) we mainly review the studies in this category.

The layered structure of interest in the current study simulated laser engagement systems known as MILES. The MILES hardware can be mounted on various firearm models and can detect when the firearm expels a blank round. It then communicates information such as field position, number of rounds fired, where the target was hit and what weapon was used among other features. Although the current MILES hardware packaging has greatly progressed from previous generations, they remain somewhat bulky and cumbersome to handle. MILES are clamped onto the rifle barrel (Figure 1) lacking realism and making laser alignment difficult.

The objective of the current investigation is to determine if the electronics used in these transmitters could be packaged, as a ‘Proof of Concept’, into a cylinder and fastened to the end of the rifle barrel (Figure 2). This would aid with laser alignment and, being mounted directly in line with the central axis of the barrel, provide a much improved level of realism.
With a functioning prototype built and the electronics tested, the next question that needs answering was how well would the design hold up in a real-world environment. In particular under mechanical shock from blank rounds as they are fired, physical abuse of the component and the impact of thermal heating on the synthetic materials all degrade the components over time.

The objective of this study is to investigate the structural vulnerabilities from a combination of stress wave dynamics through the newly-designed transmitter housing. The primary focus is to determine the structural response from high pressure, low transient gases expanding in the chamber and their effect on the components.

2 One-dimensional elastic stress waves

To simulate the shock wave effect, we adopt a simplified approach for stress wave propagation in bounded elastic media after Kolsky (1963) and Al-Haik and Almyadmeh (1993). The velocity of longitudinal waves $C_0$ in a medium of density $\rho$ and bulk modulus $K$ can be computed as

$$C_0 = \sqrt{\frac{K}{\rho}}. \quad (1)$$

This method allows computing reflection and refraction of both dilatation and distortion waves at free boundaries and at an interface between two media. We limit our discussion here to the propagation of longitudinal waves at the interface to predict the normal stresses at both sides of an interface in the composite laminate due to an incident pressure. However, the proposed approach can be extended to include torsional and lateral stress waves.

Considering a limited width $\Delta x$ of the composite layer as shown in Figure 3 with the applied stress $\sigma_{\alpha}$ and the transmitted wave producing the stress at the other side of the interface

$$\sigma_{\alpha} + \frac{\partial \sigma_{\alpha}}{\partial x} \delta x.$$
One can then write the following differential equation
\[
\rho \frac{\partial^2 u}{\partial t^2} = E \frac{\partial^2 u}{\partial x^2}.
\]  

(Figure 3) Assumptions for longitudinal wave propagation (see online version for colours)

This equation can be solved by considering a linear displacement time function \( u(t) \). That leads to realise the fundamental relationship of the stress with particle velocity as
\[
\sigma_{xx} = \rho C_0 \frac{\partial u}{\partial t}.
\]  

(Figure 4) Schematic representations of wave transmission and reflection at the interface (see online version for colours)

The term \( \rho C_0 \) is known as the characteristic impedance. Defining the particle velocity
\[
V = \frac{\partial u}{\partial t}
\]
we can re-write equation (3) to relate the particle velocity to the stress as
\[
V = \frac{\sigma_{xx}}{\rho C_0}.
\]
The above method can be extended to estimate the reflected and transmitted waves and stresses at an interface between two media. Assuming an incident stress to the interface \( \sigma_I \), part of that stress will be transmitted to the second media and part will be reflected. The transmitted stress can be computed by considering the continuity of velocity across the interface plane
\[
V_I = V_T + V_R
\]
where \( V_I \) is the incident particle velocity, \( V_R \) is the reflected particle velocity and \( V_T \) is the transmitted particle velocity. This is shown schematically in Figure 4.
Relating the incident stress to the incident particle velocity at the first layer before the interface, \( V_i \) can be computed as

\[
V_i = \frac{\sigma_i}{\rho_i C_i}. \tag{5}
\]

Equilibrium at the interface also requires that

\[
(\sigma_i + \sigma_R) A_i = \sigma_T A_i. \tag{6}
\]

By substituting equation (5) in equation (2) and solving equation (6) one can deduce the transmitted stress \( \sigma_T \) and the reflected stress \( \sigma_R \) as

\[
\sigma_T = \frac{2A_i \rho_i C_2}{A_i \rho_i C_1 + A_i \rho_i C_2} \sigma_i \tag{7}
\]

\[
\sigma_R = \frac{A_i \rho_i C_2 - A_i \rho_i C_1}{A_i \rho_i C_1 + A_i \rho_i C_2} \sigma_i. \tag{8}
\]

Of particular importance is the ratio referred to as impedance mismatches given by

\[
I = 1 - m_1 m_2. \tag{9}
\]

where

\[
m_1 = \frac{\sigma_T}{\sigma_i} = \frac{2A_i \rho_i C_2}{A_i \rho_i C_1 + A_i \rho_i C_2}; \tag{10}\]

\[
m_2 = \frac{\sigma_R}{\sigma_i} = \frac{A_i \rho_i C_2 - A_i \rho_i C_1}{A_i \rho_i C_1 + A_i \rho_i C_2}. \tag{11}\]

It is easily seen that \( 0 \leq I \leq 1 \).

The next case to be considered is the condition of a reflected stress wave at the free ends of the rod. Since the stress is assumed normal to the surface, it must be zero and opposite in sense when it is reflected (Kolsky, 1963). As an effect, the stress wave will be reflected at the free end but opposite in phase which equation (8) shows. The density of air is assumed to be zero and that will eliminate the need for equation (7) upon the stress wave reaching the free end.

### 3 Numerical experiments

For analytical purposes, the original model, see Figure 2(b), needed to be simplified so the elastic response of the structure could be studied. Features such as threads in the hardware, chamfers in the machined components exceed the capabilities of the algorithm. It would be difficult to analyse geometry of that complexity without further development and extending the scope of this research. As a result the structure was reduced to simple shapes, primarily cylinders, so that known analytical techniques could be implemented.
3.1 Materials properties

Table 1 delineates the material properties inherent in the structure used for this study. The material used for the circuit board is FR4, an epoxy based resin composite. Mechanical properties were difficult to acquire from a single source. As a result, several sources of published data from manufacturers were used to help facilitate the calculations (Corp, 2008; Matweb, 2008). For the purpose of analysis the battery was assumed to have the same mechanical characteristics of lithium. Although this assumption was crude, interest lied more in how it affected the overall results than focusing on that particular component.

Table 1  Model material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Poisson’s ratio</th>
<th>Yield strength (MPa)</th>
<th>Density (kg/m³)</th>
<th>Modulus of elasticity (GPa)</th>
<th>Acoustic speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4340 Steel</td>
<td>0.29</td>
<td>972.000</td>
<td>7850</td>
<td>205.0</td>
<td>5024</td>
</tr>
<tr>
<td>6061 Aluminum</td>
<td>0.33</td>
<td>55.200</td>
<td>2700</td>
<td>68.9</td>
<td>5675</td>
</tr>
<tr>
<td>Mild steel</td>
<td>0.29</td>
<td>330.000</td>
<td>7870</td>
<td>200.0</td>
<td>5017</td>
</tr>
<tr>
<td>304 stainless steel</td>
<td>0.29</td>
<td>215.000</td>
<td>8000</td>
<td>200.0</td>
<td>5206</td>
</tr>
<tr>
<td>FR4</td>
<td>0.12</td>
<td>0.241</td>
<td>18</td>
<td>18.6</td>
<td>3681</td>
</tr>
<tr>
<td>Lithium</td>
<td>0.36</td>
<td>4.900</td>
<td>530</td>
<td>4.90</td>
<td>3834</td>
</tr>
</tbody>
</table>

3.2 Sectioning

The input file requires a preliminary breakdown of each component into sections. Each component of the assembly could have varying cross sections. For example, the adapter has two cavities where the barrel and transmitter could be fastened on either end. In between the two cavities is a thin wall. This was used as an interface between the expanding gases in the chamber and the receiving end of the transmitter where the pressure from the expanding gases would be converted to an electrical signal. Although this was one part of the assembly, the varying cross sections would impact the calculations. For example, the outer and inner diameter of the adapter is approximately 0.017 m and 0.011 m, respectively. This results in a cross-sectional area of $1.31 \times 10^{-4}$ m². The intermediate wall cross-sectional area measured $2.87 \times 10^{-4}$ m². This component was broken down into 3 sections; one section for each of the two cavities and a third to represent the wall. This technique was applied to all assembly components.

3.3 Discretisation

The system was discretised based on the time it would take a wave to travel through one section. Wave speed propagation and lengths are different for each component and section, respectively. A numerical technique needed to be implemented to account for these differences.

The length of each section was divided by the estimated value of the wave speed and resulted in the time required for a wave to propagate through that section. The time values were divided by the lowest value calculated resulting in the number of ‘slices’ for each component. Slice values were then rounded up to the nearest integer.
For example, if a wave speed was twice as fast for component A as it was for component B, assuming the same overall length, component A would require twice the slices of component B to account for the difference in wave speed. In the case of the assembly, it was the intermediate wall of the adapter with the shortest time of $2.53 \times 10^{-7}$ s. Figure 5 displays the discretised structure.

**Figure 5**  (a) Simplified model and (b) discretised model (see online version for colours)

(a)

(b)

### 3.4 Algorithm

Microsoft Visual C++ 6.0 was used to implement the analytical equations outlined in Section 2. Once the data is read, the information is sorted into arrays, the discretisation routine is called and breaks down the components as well as its respective sections as described earlier.

The last step for the initial portion of the program was an interface index search. An interface is defined as a boundary where there is a change in cross-sectional area or a change in density. A change in either would result in a reflective component of the stress wave. The input file actually contains the boundary information in the manner it is defined, and is also taken into account in the discretisation. The index values cannot be established until the discretisation algorithm is completed. The information is stored in an array and used for post processing.

### 3.5 Boundary conditions

For the purposes of the study both ends of the structure were assumed to be in free air. One could argue the barrel requires a fixed boundary condition but the interests in this study lie with the electronic enclosure and not the barrel. Since the properties of the barrel are known, and the equations are applicable to interface interaction, the barrel is assumed to be the boundary condition to the electronic enclosure.

With the successful file input, discretisation and boundary search of the system the algorithm requires an initial stress value to start the calculation process. For the purpose of this analysis, a value of unity was input as an initial stress. It is clear from equations (7) and (8) that all the calculated values can be scaled up linearly for a different
value of incident stress. Although the net stress is important, another phenomenon of importance is that of stress amplification.

In certain scenarios the combination of wave trains transmitting and simultaneously reflecting across specific geometry could result in stress amplification (Asay and Shahinpoor, 1993). It could also result in cyclic loading of a specific part and eventually failure as a result of fatigue. A secondary input required by the user is the number of time steps. A check was incorporated into the algorithm that requires the user to enter a value equal to or greater than the twice the total number of slices. This would ensure the stress wave would run through the assembly one full cycle; all the way to the free end and back to the first slice.

4 Results and conclusions

The stress wave profiles in the system given by Figure 6 consist of steps and each step represents the arrival of a new wave at the corresponding slice. The $Y$-axis label of stress means the value of the stress wave amplitude throughout the thickness of the numbered section. Each horizontal portion of the profile indicates the time delay between arrivals of successive waves. The analytical solution, Figure 6, shows that a given wave arrives at one instant of time thereby producing a vertical segment.

Figure 6 Stress history at different times: (a) time = 104 s; (b) time = 129 s; (c) time = 150 s and (d) time = 191 s (see online version for colours)
The maximum absolute values of stress were recorded regardless if it was a compressive or expansive stress wave. The stress is assumed to be uniform over each cross sectional area. Figure 6(a)–(d) shows the summed stresses at the interfaces as the wave train propagates through the assembly for one cycle. That is, from stress initiation to the point where it reaches the free boundary. The time value also directly corresponds to the interface value for the first cycle. The barrel portion is included in the graphs. In total 9000 time steps were used for this analysis.

The time step is basically the time for the entire wave to make it through the thickness of the least thick element. Assuming a uniform square shaped pulse the duration is taken as 1 s.

The analytical solution clearly verifies that the stepped stress profiles are due to the wave reflection effects at interfaces.

Figure 7 shows the wave trains in the assembly at \( t = 132 \) s where the maximum slice stress took place. The value is 1.028 which is greater than the initial input stress of one. As a wave arrives at an interface, its ability to transmit/reflect is determined by \( m_1 \) and \( m_2 \). The impedance mismatch factor \( I \) (defined by equation (9)) determines the strength of the head wave. If \( I \approx 0 \) then most of the energy will be transmitted and the transmitted wave will be amplified; we expect a sharp rise in the stress-time curve. On the other hand, if \( I \approx 1 \) then most of the energy in the wave is spent in internal reflections resulting in a slow build up of stress. The rise (amplification of wave amplitude) characteristics are explained as the manifestation of multiple transmission/reflections occurring at the various interfaces resulting in the loss of strength of the head wave.

**Figure 7** Maximum slice stress at time = 132 s (see online version for colours)

Figure 8(a) depicts the maximum section summed stress that the search found at \( t = 2153 \) s. It is no surprise that it occurs in the first section (the section in the left side, Figure 5(a) since it contains the most slices and results in the most cumulative stress. Figure 8(b) shows the max average stress found at \( t = 116 \) s in Section 3. These two times were chosen upon examining the results archived, as they are the two times at which the maximum tensile and compressive loads were recorded for a full cycle. The stress wave went all the way to the end of the layered structure and then all the way to the point of incidence.
Figure 8  (a) Maximum section stress at $t = 2153 \text{ s}$ and (b) maximum average section stress at time $= 116 \text{ s}$

As the energy disperses the average stress will reduce, which is why the maximum average stress happened early in time, Figure 7. Hence if the design seeks to lower this maximum stress then one can choose the appropriate material combinations provided from materials properties tables.

In conclusion, an analytical solution method that captures the stress propagation in complex geometry, heterogeneous layered media has been developed. The analytical model predicts the stress rise time behaviour. The analytical results clearly support the hypothesis that layered systems will suffer stress amplification with inherent acoustic impedance mismatch between layers. It is evident that the time dependent stress response in the rise portion is significantly affected by impedance mismatch and the number of layers of different materials.

Although the components of the MILES system were simplified, the analytical solution does provide insight into what could be happening in the structure. The rise of stress higher than the initial induced stress could be a cause for concern especially
if the incident stress is in the vicinity of the yield or ultimate tensile strength of any of the layered heterogeneous materials.

The analytical model can provide a more economical alternative to commercial explicit Finite Element Analysis (FEA) codes – most notably LS-DYNA (Hallquist, 2006) – that can account for some of the complex geometry.

References


